

MONTHLY NOTICES
OF THE
ROYAL ASTRONOMICAL SOCIETY

Vol. 98 No. 6

MEETING OF 1938 APRIL 8

Dr. H. Spencer Jones, President, in the Chair.

The election by the Council of the following Fellows was duly confirmed:—

Ronald George Foster, M.Sc., 261 Kent House Road, Beckenham, Kent;

Rev. H. Hancock, M.A., F.R.S.A., F.R.Met.Soc., 88 Heene Road, Worthing;

John Robert Fisher Jeffreys, Downing College, Cambridge;

Charles William Bloomfield Michie, Box 107, Kaitaia, New Zealand;

Thomas Silcock, B.A., Heap Lynn, Seaton Lane, West Hartlepool, Co. Durham;

Wen-Sai Tai, Gonville and Caius College, Cambridge; and

Ivan Leslie Thomsen, Dominion Observatory, Wellington, New Zealand.

The following candidates were proposed for election as Fellows of the Society, the name of the proposer from personal knowledge being appended:—

Canon Edward Baunton Ford, Chancellor of the Diocese of Bloemfontein and Rector of Wepener, O.F.S., South Africa (proposed by R. A. Rossiter);

Robert R. McMath, Director of the McMath-Hulbert Observatory, Lake Angelus, Michigan, U.S.A. (proposed by W. S. Adams); and

Mentore Maggini, Director, R. Osservatorio Astronomico, Collurania-Teramo, Italy (proposed by T. E. R. Phillips).



The President announced that the following Associates had been elected by the Council:—

Monsieur L. d'Azambuja, Observatoire de Meudon, France;
 Professor A. O. Leuschner, University of California, Berkeley, Cal.,
 U.S.A.; and
 Professor Knut Lundmark, Director of the Lund Observatory, Sweden.

Eighty-seven presents were announced as having been received since the last meeting, including:—

Hamburg Observatory: Bergedorfer Spektral Durchmusterung, Band 2.

The President announced that Professor Charles Fabry would deliver the George Darwin Lecture at the Ordinary Meeting of the Society on May 13, taking as his subject "Interstellar Space."

Mr. Essen, of the National Physical Laboratory, spoke on quartz crystal clocks.

At a Special General Meeting of Fellows held before the Ordinary Meeting, the following amendments to the bye-laws, which were recommended by the Council as being necessary to bring the wording of the bye-laws in question into conformity with other bye-laws which have been altered at General Meetings of Fellows in recent years, were proposed by the Secretaries and carried unanimously:—

Bye-law.

Amendment.

No. 26. Whenever a Fellow of the Society shall be one year in arrear in the payment of his annual contribution he shall cease to enjoy the right of attending any meeting of the Society, of assuming the title of a Fellow of the Society, and of receiving the publications of the Society and other advantages thereof until his arrears be paid; and should such Fellow become two years in arrear as aforesaid, his name shall be publicly suspended as a defaulter in the meeting-room at any ordinary meeting of the Society; and should his arrears be still unpaid at the next ordinary meeting of the Society (or in the case of Fellows resident abroad, at such later date as the Council shall determine), and no satisfactory reason assigned to the Council for such non-payment, he shall cease to be a Fellow of the Society.

For "publicly suspended as a defaulter in the meeting-room"
read "publicly exhibited as that of a defaulter in the Library (or in such other place as the Council may from time to time direct)."

No. 28. Every such person, proposed for admission as an Associate, must be recommended by three or more Fellows; all of whom must certify in writing, agreeably

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to the Form No. 4 in the Appendix hereto, that he is a person eminent in the science of Astronomy, and that they have a personal knowledge of him, or are acquainted with his works.

Delete.

No. 29. The election of Associates shall rest with the Council.

Re-number as bye-law 28.

Nos. 30-45 inclusive.

Re-number as 29-44.

No. 45A.

Re-number as 45.

No. 43. The business of the ordinary meetings shall be to propose and ballot for Fellows and Associates; to announce such donations as may have been made to the Society; to read such communications relating to Astronomy and the subjects connected therewith, as may have been presented to the Society; and to proceed upon any other subject which may have been authorised by the Council: provided always that the same do not relate to the Bye-laws, nor to the management of the Society.

Delete "and Associates."

Appendix. Form No. 3, copy of bye-law 26 to be enclosed.

Alter wording of bye-law 26 as above.

Appendix. Form No. 4.

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COLOUR TEMPERATURE VARIATIONS OF γ CASSIOPEIÆ, AND THE PROBLEM OF THE "YELLOW" B-TYPE STARS.

W. M. H. Greaves and E. Martin.

(Communicated by the Astronomer Royal)

During the summer of 1936 a striking increase took place in the brightness of the star γ Cassiopeiæ, the Harvard spectral type for which is Bop. The normal visual magnitude of this star is 2^m.25, but by 1936 August it had brightened to 1^m.8, and it subsequently reached 1^m.6 in 1936 October. The star was put on to the Greenwich colour temperature observing programme in 1936 November. Observations for colour temperature had previously been secured in 1926 and 1927, and the details of these observations were given in a Greenwich publication which was issued a few years ago.* The gradients measured from the plates taken in 1926 and 1927 are repeated in the table below, together with the gradients derived from plates taken in 1936 and 1937.

The table gives the gradients G defined by the equations

$$G = \phi - \phi_0,$$

$$\phi = \frac{c_2}{T} \left(1 - e^{-2c_2/T} \right),$$

where T is the colour temperature, $c_2 = 14320$, and the zero suffix refers to the zero-point of the Greenwich standard system, defined as the mean of 9 stars of type Ao. The plates taken in 1926 and 1927 were secured with the 30-inch reflector, used in conjunction with a slitless spectrograph which incorporated a 6-inch triplet lens of 27 inches focus. This arrangement gave tolerably good definition near wave-lengths 4100 Å. and 5600 Å., and the spectra were measured at eight points in the blue extending from 3994 Å. to 4204 Å., and at eight points in the yellow extending from 5385 Å. to 5789 Å. During this early work the precise wave-lengths used were readjusted several times. They were selected so as to be clear of spectral lines.

The 1936 and 1937 plates were taken with the Yapp 36-inch reflector, used in conjunction with a reflecting slitless spectrograph similar in construction (but incorporating some improvements in detail) to that used with the 30-inch reflector from 1928 February onwards, and described in the Greenwich publication referred to above.† These spectra were measured at eight wave-lengths in the blue extending from 4155 Å. to 4509 Å., the harmonic mean being 4284 Å., and at eight wave-lengths in the red extending from 6082 Å. to 6486 Å., the harmonic mean being 6320 Å. The wave-lengths used were selected so as to be clear of spectral lines, and the values of G given in the table were derived from these measures. A few

* *Observations of Colour Temperatures of Stars*, 1932.

† *Loc. cit.*, pp. 11 and 12.

of the plates were also measured (see below) at four wave-lengths in the green extending from 5234 Å. to 5382 Å., with a harmonic mean at 5308 Å. It may be mentioned that the spectrograph was designed for use over the spectral range $H\delta$ to $H\alpha$. For wave-lengths in the infra-red and ultra-violet the beam is no longer full, being cut off by the edge of the diagonal flat.

Values of G

Date	G	Date	G
1926 May 15	-0.09	1936 Nov. 9	+0.56
May 16	-0.01	Nov. 21	+0.46
May 31	+0.01	Dec. 4	+0.50
Oct. 7	+0.03	Dec. 6	+0.33
Oct. 9	+0.13		
Oct. 31	+0.31	1937 Sept. 3	+0.57
Nov. 5	-0.10	Sept. 4	+0.60
1927 Jan. 10	+0.24	Sept. 26	+0.67
Jan. 25	+0.03	Sept. 29	+0.69
Jan. 26	+0.11	Oct. 4	+0.64
Jan. 27	+0.06		

The comparison stars used in 1926 and 1927 were α Aquilæ, ϵ Persei, η Tauri and η Ursæ Majoris. For the 1936 and 1937 plates the comparison star was δ Cassiopeiæ. The gradients adopted for these stars are those given in the 1932 Greenwich publication. The probable errors of the adopted values of G for the comparison stars are ± 0.01 for α Aquilæ, η Tauri, η Ursæ Majoris and δ Cassiopeiæ, and ± 0.02 for ϵ Persei.

Each value of G for γ Cassiopeiæ was obtained from a spectrophotometric comparison in which γ Cassiopeiæ and the comparison star were photographed at approximately equal altitudes. Small corrections for differential atmospheric reddening and for inequality of exposure time* have been applied. The average corrections for atmospheric reddening are +0.027 for 1926 and 1927, +0.005 for 1936 and +0.008 for 1937. The corresponding average corrections for time effects are -0.003, +0.005 and +0.028.

The 1926 and 1927 plates were calibrated with a coarse objective grating, and those taken in 1936 and 1937 were calibrated with a multiple slit spectrograph.

All the plates were measured in a visual microphotometer (described on pp. 15 and 16 of the Greenwich 1932 publication) by two measurers.†

On taking the mean values of G for 1926–1927, 1936 and 1937, we obtain the following:—

Mean Date	G	T
1926.78	+0.07	16,100°
1936.90	+0.46	10,500°
1937.72	+0.63	9,200°

* *M.N.*, 96, 825, 1936.

† Some preliminary measures of the 1936 plates were made by E. Vandekerckhove whilst on a visit to the Royal Observatory, and published in *The Observatory*, 60, 329, 1937.

The values of the colour temperature T are based on the value $\phi_0 = 1.00$ (see *M.N.*, 94, 505, 1934). They are obtained by adding 1.00 to the values of G and using the conversion table given on p. 63 of the Greenwich 1932 publication.

The above values show a well-marked decrease of colour temperature between 1926.8 and 1936.9, and a further decrease between 1936.9 and 1937.7. As regards the first decrease, it may perhaps be argued that it is due to the different wave-lengths employed in the measurements at these two epochs. Such an explanation is *a priori* unlikely, experience having shown that over the whole visual range when stars of types B and A are compared, the plots of magnitude difference against reciprocal of wave-length are linear within the error of measurement, so that the measured differences of gradient are independent of the exact wave-lengths used. γ Cassiopeiæ, however, is not a normal star, and it may be considered conceivable that its spectral distribution of energy should be abnormal in this respect. To test this point three of the later plates were measured at four wave-lengths in the green, the mean harmonic wave-length being 5308 Å. (see above).^{*} The following table gives the residuals at 5308 Å. from the linear gradients derived from the measures in the blue and red. The residuals are given in magnitudes, a positive sign denoting that, relative to the comparison star, the energy output of γ Cassiopeiæ at 5308 Å. is in excess of that given by a linear gradient which has been fitted to the blue and red measures.

Residuals at 5308 Å.

Date	Residual m
1936 Nov. 9	+0.04
1937 Sept. 29	-0.08
1937 Oct. 4	+0.03

These residuals are regarded as accidental. It is clear that there is no large departure in the case of γ Cassiopeiæ from the normal linear gradient (corresponding very closely to a Planck distribution of energy) throughout the visible region of the spectrum.

It seems quite certain that there was a large increase in the gradient of γ Cassiopeiæ (corresponding to a decrease in colour temperature) between 1927 and the autumn of 1936. The change probably took place in the summer of 1936 at the time that the star brightened. Chalonge and Safir † have drawn attention to a decrease in the colour temperature of γ Cassiopeiæ between 1935 September and 1936 November. The change of colour temperature given by Chalonge and Safir corresponds to an increase of 0.24 in gradient, but it is stated that the 1936 results are rough. The values given in the table above suggest a somewhat greater change.

^{*} The emulsions used are relatively insensitive in the green, and very often the spectra are unsuitable for measurement in this region. The three plates in question had been exposed rather more heavily than the average and were selected for this reason.

† *C.R.*, 203, 1329, 1936.

It is probable that the increase of gradient of γ Cassiopeiae in 1936 and the further increase, as shown in the table, between the autumn of 1936 and that of 1937, are related to the intensities of the emission lines of hydrogen. Chalonge and Safir, in the paper referred to above, have drawn attention to the fact that the brightening of the star in 1936 was accompanied by increases in the intensities of the hydrogen emission lines relative to the continuous spectrum, and of the continuous emission beyond the Balmer limit. On the Greenwich plate of 1936 November 9, which was exposed sufficiently to bring up the near ultra-violet, 13 lines of the Balmer series can be seen in emission. An inspection of the Greenwich plates for 1936 and 1937 shows that by the autumn of 1937 a further strengthening of the emission lines had taken place. In order to check this purely qualitative conclusion the contour of $H\gamma$ was measured on the plates for 1936 December 6 and 1937 September 26. The resulting total intensities are given in the following table, the actual quantities measured being the areas between the measured contours and the interpolated continuous. The unit is in each case the output of energy in one angstrom of the continuous spectrum. The corresponding intensity of $H\gamma$ for 1927 January 10 was also measured, but, owing to optical imperfections (see above), the result for this date is relatively inaccurate.

Total Intensities of $H\gamma$

(Unit = radiation in 1 Å. of continuous)

Date	Intensity
1927 Jan. 10	(1.0)
1936 Dec. 6	2.8
1937 Sept. 26	5.5

These figures confirm the qualitative conclusions stated above. The progressive decrease of the colour temperature of γ Cassiopeiae in 1936 and 1937 seems to be accompanied by a progressive strengthening of the intensities of the emission lines.

The change of colour temperature places γ Cassiopeiae definitely in the category of "yellow" B-type stars. In 1923 Hertzsprung * drew attention to the existence of this group of stars. In 1928 Greenwich colour temperature results for some of these stars were published,† and it was pointed out that the colour temperatures appeared to be correlated with the intensities of the interstellar K line, the stars with strong interstellar lines having relatively low colour temperatures. On the strength of this the Greenwich observers suggested that the reddening effect was due to selective absorption or scattering in interstellar space. This suggestion was criticised in 1929 by Gerasimovič,‡ who pointed out that some of the "yellow" B-type stars were characterised by enhanced emission (or diminished absorption) on the ultra-violet side of the Balmer series limit; he concluded that the low colour temperatures were intrinsically connected with the more luminous stars of type B.

* *B.A.N.*, I, 204, 217, 1923.

† *M.N.*, 89, 125, 1928.

‡ *Harvard Circular* 339, 1929.

Gerasimovič also pointed out that statistically the B-type stars with emission lines have lower colour temperatures than normal B stars with absorption spectra.

The hypothesis of space reddening was again advanced in 1930 by Trumpler,* who put forward weighty reasons for the existence of total and selective space absorption. In 1934 E. G. Williams† examined afresh the relationship between colour excess and the intensity of the interstellar K line. On plotting colour excess as ordinate against total absorption in K as abscissa, Williams found a marked correlation for B-type stars with absorption spectra, the stars with strong K showing relatively strong colour excess, but he also found that the emission B stars fell on the diagram above the line drawn through the representative points of the stars with absorption spectra. Furthermore, Williams showed that the residual colour excess (after allowing for distance effect) of Be stars seemed to be related to the intensity of the hydrogen emission, the stars with stronger emission lines being, on the average, lower in colour temperature.

Williams's work suggests that over and above a possible reddening effect in space there is a reddening for the Be stars which must be an intrinsic property of these stars. The results for γ Cassiopeiae given in the present paper are in entire accordance with this conclusion. The fall in colour temperature of γ Cassiopeiae in 1936 and 1937 cannot be ascribed to selective absorption in interstellar space, and the accompanying increase of the intensities of the emission lines, when considered in connection with Williams's results, leads us to seek for some physical mechanism which will produce emission lines, and which will also redden the continuous spectrum in the visual region. It may be noticed at this point that the colour temperature of Nova Herculis 1934 for the date of discovery, when the emission spectrum was definitely early B type, was that corresponding to a normal A-type star.‡ This again is not a space-reddening effect, for a few days later the colour temperature was in good agreement with that normally associated with the corresponding spectral type.

It is possible that, for some reason at present unknown, Be stars tend to possess extended atmospheres, and that the lowering of colour temperature is due to selective scattering in the outer chromosphere. But a suggestion due to Woolley§ deserves attention and further investigation. Woolley suggests that the low colour temperatures of the yellow stars of type B are the consequence of fluorescent emission, extending throughout the visible spectrum, and coming to a maximum at the head of the Paschen series. It would be expected qualitatively that such a continuous emission would be accompanied by line emission (or by a weakening of line absorption) in the Balmer series, and this would fit in very well with the facts as at present known.

Further theoretical and observational studies of the emission B-type

* *L.O.B.*, 420, and *P.A.S.P.*, 42, 214, 267, 1930.

† *Ap. J.*, 79, 280, 1934. Reprinted as *Mount Wilson Contribution*, No. 487.

‡ *The Observatory*, 60, 67, 1937.

§ *M.N.*, 95, 115, 1934.

stars are needed. Williams (*loc. cit.*) could only discriminate qualitatively between Be stars with strong and weak emission lines, and it would be interesting to undertake a detailed quantitative study of the correlation between the colour temperatures of Be stars and the intensities of the Balmer lines. A systematic spectrophotometric measurement of the line intensities seems to be very desirable. Further quantitative measurements of the strength of the continuous spectrum for wave-lengths less than the Balmer limit, such as have been carried out by Arnulf, Barbier, Chalonge and Canavaggia,* would also be helpful in view of Gerasimovič's qualitative conclusion that "yellow" stars of type B tend to exhibit enhanced emission in this spectral region.

Summary.—Colour temperature results are given for γ Cassiopeiæ. The brightening of the star in 1936 was accompanied by a strengthening of the emission lines and by a fall in the colour temperature. Between the autumns of 1936 and 1937 a further increase of the strength of the emission lines and a further decrease of the colour temperature took place. The general problem of "yellow" stars of type B is discussed. The available evidence indicates that in addition to a possible reddening due to selective space absorption, there is a reddening effect which is intrinsic in the stars, and which seems to be associated with the development of line emission.

Royal Observatory, Greenwich :

1938 March 16.

* *Journal des Observateurs*, 19, 149, 1936.

THE DETERMINATION OF THE CONSTANT OF NUTATION FROM THE GREENWICH LATITUDE VARIATION OBSERVATIONS.

H. Spencer Jones, F.R.S., Astronomer Royal.

A determination of the constant of nutation from the latitude variation observations obtained with the Cookson floating telescope at Greenwich in the period 1911–1929 was made by J. Jackson * in 1930. The programme of observations was continued until the middle of 1936, when the site of the instrument was changed and a new programme was commenced. The observations obtained in the last seven years give additional material for the derivation of the constant of nutation. A redetermination of this constant, using all the material available from this programme, seemed to be desirable.

The method employed by Jackson was closely followed. The latitude variation observations give

$$Z.D. \text{ north} = \delta - \phi,$$

and δ can be put into the form $\delta = \delta_0 + \mu t + \text{periodic terms}$, where the periodic terms include nutation. By comparison of the observations on the same nights of evening and morning groups, we have a comparison between groups with different nutation factors, the latitude (which is a variable quantity) being eliminated.

For the discussion of the constant of nutation certain small corrections need to be applied to the data. The aberration applied in the reductions does not include the perturbations of Jupiter and Saturn, which have periods of 12 and 30 years respectively. These were applied in the form given by Battermann,†

$$C[\cos \lambda (\cos \epsilon \sin \delta \sin \alpha - \sin \epsilon \cos \delta) - \sin \lambda \sin \delta \cos \alpha],$$

where λ is the heliocentric longitude of the planet, and $C = 0''.0086$ for Jupiter and $0''.0019$ for Saturn.

In addition there are some small terms in the nutation that are usually neglected but which must be taken into account. These terms were tabulated by Ross ‡ for the interval 1900–1920. Jackson § continued this tabulation for the interval 1920–1929. For completeness, and because these data are of use in any investigation of the constant of nutation covering this interval, the additional terms are here tabulated for the interval 1930–1936.

* *M.N.*, 90, 733, 1930.

† Resultate aus den Polhohenbestimmung in Berlin, *Central Bureau der Internationalen Erdmessung*, 1899.

‡ *Astr. Nach.*, 192, 47, 1912.

§ *M.N.*, 90, 733, 1930.

TABLE I

Values of $\sin \epsilon \delta \lambda$ Neglected in National Ephemerides. Unit ".001

Fraction of Year	1930	1931	1932	1933	1934	1935	1936
.00	+1	-2	+1	+1	-4	-6	-2
.03	+1	-2	+1	+2	-3	-5	-1
.06	+1	-2	0	+2	-2	-3	+1
.09	0	-3	-1	+1	-1	-2	+2
.12	-1	-4	-2	0	-1	-1	+2
.15	-3	-5	-4	-2	-2	-1	+2
.18	-4	-5	-4	-3	-2	-1	+2
.21	-5	-5	-4	-4	-3	-1	+2
.24	-4	-4	-3	-4	-3	0	+3
.27	-2	-1	0	-2	-2	+2	+4
.30	+1	+2	+3	0	-1	+2	+5
.33	+3	+5	+5	+2	-1	+3	+5
.36	+4	+6	+7	+3	-1	+2	+5
.39	+5	+7	+8	+3	-2	+1	+4
.42	+4	+5	+6	+2	-4	-2	+2
.45	+2	+3	+4	0	-6	-5	-1
.48	-1	0	+2	-2	-9	-7	-3
.51	-3	-3	-1	-4	-10	-9	-4
.54	-5	-6	-3	-5	-11	-10	-5
.57	-6	-7	-5	-6	-10	-9	-5
.60	-6	-7	-5	-5	-8	-7	-3
.63	-5	-5	-4	-3	-5	-3	0
.66	-2	-3	-2	-1	-1	+1	+4
.69	0	0	+1	+1	+2	+5	+6
.72	+2	+3	+3	+3	+4	+8	+8
.75	+4	+5	+4	+4	+5	+9	+8
.78	+5	+6	+5	+3	+5	+9	+8
.81	+4	+7	+5	+2	+3	+7	+6
.84	+3	+6	+4	0	0	+5	+3
.87	+2	+5	+3	-2	-2	+3	+1
.90	+1	+4	+2	-3	-5	-1	-1
.93	0	+3	+2	-4	-6	-2	-2
.96	-1	+2	+2	-5	-7	-2	-1
.99	-1	+2	+2	-4	-6	-2	0
1.02	-2	+1	+1	-3	-6	-1	+1

TABLE II

Values of $\delta \epsilon$ Neglected in the National Ephemerides. Unit ".001

Fraction of Year	1930	1931	1932	1933	1934	1935	1936
.00	+5	+2	0	+3	+2	-4	-5
.03	+7	+3	+1	+3	+1	-5	-5
.06	+9	+4	+1	+3	+1	-5	-5
.09	+9	+3	0	+2	0	-5	-4
.12	+8	+2	-1	+1	-1	-5	-3
.15	+6	-1	-3	0	-2	-5	-2
.18	+3	-4	-5	-1	-3	-5	-1
.21	-1	-7	-7	-3	-3	-5	0
.24	-4	-9	-9	-3	-3	-4	+2

TABLE II—*continued*

Fraction of Year	1930	1931	1932	1933	1934	1935	1936
.27	-6	-11	-9	-3	-3	-3	+3
.30	-7	-11	-9	-2	-2	-2	+4
.33	-6	-9	-6	0	0	0	+5
.36	-4	-6	-3	+3	+3	+2	+7
.39	0	-3	0	+6	+5	+3	+7
.42	+3	0	+3	+8	+6	+4	+7
.45	+6	+3	+5	+9	+6	+3	+6
.48	+8	+4	+6	+9	+5	+2	+5
.51	+8	+3	+5	+6	+2	-1	+2
.54	+6	+1	+2	+3	-2	-5	-2
.57	+4	-2	-2	0	-6	-8	-4
.60	+1	-5	-5	-4	-9	-10	-5
.63	-2	-8	-7	-6	-10	-11	-6
.66	-5	-11	-9	-7	-11	-11	-4
.69	-7	-11	-9	-7	-10	-8	-2
.72	-6	-10	-7	-4	-6	-4	+2
.75	-5	-8	-4	0	-2	0	+7
.78	-4	-6	0	+3	+2	+4	+10
.81	-2	-3	+3	+6	+4	+7	+12
.84	-1	-1	+5	+8	+6	+8	+12
.87	0	0	+6	+9	+5	+7	+10
.90	+1	+1	+7	+9	+5	+5	+8
.93	+2	+2	+6	+7	+2	+3	+4
.96	+2	+1	+5	+5	-1	-1	0
.99	+2	0	+4	+2	-4	-4	-4
1.02	+2	0	+3	+1	-5	-4	-4

This investigation has been based on revised star places and proper motions, details of which will be given later in a Greenwich publication. Corrections for scale and temperature have been applied to the residuals between the observed and tabular places. The small additional terms mentioned above were also applied. The mean residuals, so corrected, for each star-pair and epoch were collected.

Changes in the list of stars under observation were made in 1919 and in 1929. For the derivation of the constant of nutation the following material is available :—

- (a) 43 star-pairs remained on the programme for the whole period of observation, 25 years.
- (b) 22 star-pairs were observed from 1919 (or 1920) to 1935 (or 1936), *i.e.* for nearly a complete nutation period.
- (c) 4 star-pairs were observed from 1911 to 1928 (or 1929) and were then removed from the programme.

The material was therefore discussed in three sections, *viz.* :

- (i) Using all star-pairs observed from 1911 to 1928.
- (ii) Using all star-pairs observed from 1919 to 1935.
- (iii) Using all star-pairs observed throughout.

The means of the residuals for the star-pairs observed at each epoch were formed, equal weight being given to each pair irrespective of the

number of observations. Evening and morning observations were treated separately. The corresponding mean epochs and mean nutation factors were also formed. The nutation factors were computed from the formula :

$$N = +0.744 \cos \alpha \sin \Omega - 1.000 \sin \alpha \cos \Omega \\ - 0.119 \cos \alpha \sin 2\odot + 0.129 \sin \alpha \cos 2\odot.$$

The residuals so obtained were corrected for wind effect and for the diurnal variation of latitude. These have been deduced from the discussion of the 25 years' observations. The wind effect can be represented by a term $0''.054 \cos(\theta - 28^\circ.3)$, where θ denotes the direction of the wind, measured from north through east, which causes an apparent displacement of the zenith to the south when the wind is from the N.N.E. and to the north when the wind is from the S.S.W. The diurnal variation of latitude produces a term in the residuals of amount $-0''.015 \sin(\odot - \alpha)$, causing a systematic difference between evening and morning residuals.

The mean epoch for evening and morning pairs observed during the same period is not always the same. Where the epochs were different, the curve for the variation of latitude was used to reduce the morning observations to the epoch of the evening observations, as the method of reduction used to derive the constant of nutation assumes that latitude variation is eliminated when the evening and morning observations are differenced.

For each combination of evening and morning groups, the differences between evening and morning residuals were analysed in the form $\alpha + \beta t + \gamma N$, where N is the difference between evening and morning nutation factors and γ is the correction to the nutation constant. The analysis was made by a least-squares solution, the weight assigned to a combination of a evening pairs with b morning pairs being $ab/10(a+b)$.

With this system of weighting the equations for γ obtained after elimination of α and β are given in Table III. The results are given for each separate combination of evening and morning groups for the three separate periods. The number of evening and morning pairs utilised are also given, as well as the probable error corresponding to unit weight derived from the solution for the full period, 1911-1936.

The several determinations of the probable error for unit weight are reasonably concordant. We may take as the best value for the probable error the mean of the determinations, $\pm 0''.052$.

If ϵ represents the probable error of the mean zenith distance derived from a single star-pair, the probable error of the difference of the mean residuals from a total of a morning pairs and of b evening pairs would be

$$\epsilon \sqrt{\left(\frac{1}{a} + \frac{1}{b}\right)},$$

provided that the number of observations of each morning pair or of each evening pair were equal. In order that the quantities α and β in the solutions should represent correction to the mean star-places and to the mean

proper motions of the stars considered, it was necessary to give the same weight to each pair in an evening or morning combination for any one epoch, although the number of observations of the various pairs may differ. In general, the differences in the numbers are not large, but the effect of any inequalities in the numbers will be to make the probable error corresponding to unit weight rather greater than it would otherwise have been.

TABLE III

Results for Constant of Nutation

Groups <i>E</i> <i>M</i>	1911-1928		1920-1936		1911-1936		P.E. Unit Weight
	Equation for γ	No. of Pairs <i>E</i> ; <i>M</i>	Equation for γ	No. of Pairs <i>E</i> ; <i>M</i>	Equation for γ	No. of Pairs <i>E</i> ; <i>M</i>	
14, 15 : 2, 3	24.77 $\gamma = -.563$	8 ; 5	15.69 $\gamma = -.835$	9 ; 6	41.52 $\gamma = -.602$	7 ; 4	$\pm .052$
16, 1 : 4, 5	15.33 .000	6 ; 4	22.51 +.949	7 ; 8	17.94 +.546	5 ; 4	$\pm .053$
2, 3 : 6, 7	5.35 +.446	5 ; 6	9.96 +.205	6 ; 8	20.77 +.928	4 ; 5	$\pm .063$
4, 5 : 8, 9	6.48 +.028	4 ; 4	12.06 -.265	8 ; 8	21.21 +.219	4 ; 4	$\pm .059$
6, 7 : 9, 10	3.69 -.062	6 ; 4	6.85 -.205	8 ; 9	6.14 -.010	5 ; 4	$\pm .039$
8, 9 : 10, 11	2.83 +.076	4 ; 6	3.68 +.253	8 ; 9	4.39 +.155	4 ; 6	$\pm .037$
12, 13 : 14, 15	0.66 +.054	8 ; 8	1.10 +.066	10 ; 9	2.95 +.119	8 ; 7	$\pm .055$
13, 14 : 16, 1	5.97 -.334	8 ; 6	5.26 +.046	10 ; 7	17.75 -.144	8 ; 5	$\pm .055$

If ϵ_0 is the probable error corresponding to the unit of weight that we have adopted in the solutions for γ , we have

$$\epsilon = \sqrt{10 \times \epsilon_0}.$$

If ϵ_p is the probable error of a single plate, which on the average contains about 4.5 star-pairs, we have

$$\epsilon = \sqrt{4.5 \times \epsilon_p}.$$

We infer that, with $\epsilon_0 = \pm ".052$,

$$\begin{aligned} \epsilon_p &= 1.49\epsilon_0 \\ &= \pm ".078. \end{aligned}$$

This is exactly the probable error for a single plate derived in the Greenwich volume containing the discussion of the Cookson telescope observations for 1911-1919. A rather smaller result was anticipated, as allowance has been made for wind-effect in the reductions. On the other hand, the tendency is for ϵ_0 to be estimated too great, as a result of the inequalities in the number of observations of different pairs.

The conclusion is that a value of $\pm ".052$ for the probable error of the unit of weight adopted in the reductions is not likely to be an underestimate.

Adopting this value, the material for the three periods gives the following corrections to the constant of nutation :—

1911-1928	-".0054 ± ".0065
1920-1936	+".0028 ± ".0059
1911-1936	+".0091 ± ".0045

If these three determinations were entirely independent, their combination would give as the correction to the constant of nutation the value $+".0039 \pm ".0032$. But they are not entirely independent. Inspection of the number of star-pairs used, given in Table III, shows that the material used for the period 1911-1928 is largely included in the complete period 1911-1936, and that the first period contributes very little additional weight. The solution for the complete period gives the corrections to the mean proper motions with the greatest weight and, because of the partial correlation between the proper-motion term and the nutation term, when a complete nutation period is not fully covered, the solution for the whole period is most likely to be free from errors that may be introduced by such partial correlation.

The second period contains a considerable number of stars that were introduced into the programme in 1920, though more than half the material for this period is also incorporated in the solution for the full period.

The results were therefore combined in the following way. The relative weights, so far as the number of evening and morning pairs used are concerned, were determined. For the first combination, the relative weights are 12.3, 14.4 and 10.0. In the first period, of the total weight of 12.3, the material corresponding to a weight of 10 is included in the full period and should not be weighted twice; the equation for γ for the first combination and first period was therefore multiplied by a factor of 0.187 (equivalent to $2.3/12.3$). The corresponding equation for the second period was multiplied by 0.306 (equivalent to $4.4/14.4$). The two equations, so reduced, were added to the equation for the full period.

Each combination was dealt with in this way. The final equation for γ was

$$170.17\gamma = +1.250$$

corresponding to

$$\gamma = +0''.0073 \pm 0''.0040.$$

The value of the constant of nutation is therefore, since the value used in the ephemerides is $9''.210$,

$$9''.2173 \pm 0''.0040.$$

Newcomb used meridian observations, mainly those made between 1850 and 1895 at Greenwich, Pulkovo and Washington, to derive the constant of nutation, and obtained the value $9''.210 \pm 0''.008$. The value that he inferred from the precessional constant and the mass of the Moon was $9''.214$. Przybyllok from 15 years' observations (1900-1915) at the five stations of the International Latitude Service obtained the value $9''.207 \pm 0''.003$. The discussion of the Cookson observations for 1911-1929 by

Jackson gave the value $9''.2066 \pm 0''.0055$. The value derived above for the first period differs slightly from Jackson's value, because of the revision of the star-places and proper motions, the application of wind and diurnal latitude variation corrections, and the reduction of morning and evening observations to identical epochs. Jackson incorporated observations in 1929 after some of the pairs had been dropped from the programme; this caused the weight of his determination to be somewhat greater than that of our first period.

Jackson remarked on the discordance between the theoretical value of the constant of nutation and the observed value, which he regarded as "one of the outstanding discordances of the constants of the solar system." The value derived in the present discussion approaches much more nearly to the theoretical value. It is desirable that a more complete discussion of the material available in the International Latitude results should be made for the derivation of the constant of nutation. The discussion by Przybyllok did not cover a full nutation period, and used the observations only up to the year 1915.

Newcomb * gives the following relations between p , the luni-solar precession; N , the constant of nutation; μ , the mass of the Moon; and $(C-A)/C$, the mechanical ellipticity of the Earth:—

$$p = [5.975052] \cos \epsilon \frac{1}{1 + \mu^{-1}} \frac{C-A}{C} + [3.72509] \cos \epsilon \frac{C-A}{C},$$

$$N = [5.40289] \cos \epsilon \frac{1}{1 + \mu^{-1}} \frac{C-A}{C}.$$

He remarks that, in these formulæ, "the numerical coefficients may be regarded as absolute constants, so accurately determined that no question of their errors need now be considered."

Using the obliquity for 1900, the following table gives the reciprocal of the mass of the Moon corresponding to different values of the nutation constant and of the luni-solar precession:—

p	$N=9''.20$	$N=9''.21$	$N=9''.22$
"			
50.36	81.86	81.58	81.30
50.37	81.91	81.63	81.35
50.38	81.96	81.68	81.40
50.39	82.02	81.73	81.45
50.40	82.07	81.78	81.50

Fotheringham pointed out that the relativity effect on precession had previously been applied with the wrong sign, and this was confirmed both by de Sitter and Eddington.† The true luni-solar precession for 1900 is therefore $50''.3899$, according to Newcomb, or $50''.4012$, according to Oort.‡

* *Astronomical Constants*, p. 132.

† See addendum (p. 742) to Jackson's paper, *M.N.*, 90, 733, 1930.

‡ *B.A.N.*, 4, No. 132, 79, 1927.

The latter investigation took into account the rotation of the galaxy, and a closely concordant value was obtained by Fotheringham.*

The values of the reciprocal of the mass of the Moon corresponding to Newcomb's and Oort's values of the luni-solar precession and to three values of the constant of nutation, viz. $9''.207$ (Przybyllok's value), $9''.210$ (Newcomb's adopted value), $9''.217$ (the value derived in the present investigation), are given in the following table :—

$\frac{p}{N}$	$50''.3899$	$50''.4012$
"		
9.207	81.82	81.87
9.210	81.73	81.79
9.217	81.53	81.59

The reciprocal of the mass of the Moon derived by Hinks from the observations of Eros at the 1901 opposition was $81.53 \pm .05$. This mass supports a larger value of the constant of nutation than has hitherto been accepted, whichever value of the precession is adopted. The observations of Eros at the 1931 opposition should give a more accurate determination of the mass of the Moon than that given by the 1901 observations, and should prove of the greatest value for the more accurate determination of the nutation constant.

Summary.—The material available in twenty-five years' observations (1911–36) with the Cookson floating zenith telescope at the Royal Observatory, Greenwich, has been analysed for the determination of the constant of nutation. The value obtained for this constant is $9''.217 \pm 0''.004$. The inferred value for the reciprocal of the mass of the Moon is 81.53 , on the basis of Newcomb's value of the luni-solar precession, or 81.59 , on the basis of Oort's value, which takes galactic rotation into account.

* *M.N.*, 86, 414, 1926.

ON THE MOTION OF THE APSIDAL LINE IN CLOSE BINARY SYSTEMS.

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1. *Introduction.*—The orbits of close binaries is one of the most attractive problems of dynamical astronomy. As soon as the dimensions of the components become comparable with their relative distances, the components become distorted in form by the effect of their mutual attraction, and their orbits cease to be simple conics, for obvious reasons. The question concerning the nature of departures of their orbits from initial ellipses caused by the elliptical form of the components has already been a subject of several recent investigations. S. Tscherny,* in the special case of the β Lyræ system, and H. N. Russell,† in a more general investigation, came to the conclusion that the orbits can essentially be represented by Keplerian ellipses with a secular advance of periastron. K. Walter,‡ on the other hand, claimed to have shown that in the case of a constrained rotation, *i.e.* if the periods of the axial rotation of both components and that of their revolution agree, a recession of the apsidal line takes place.§ As the line followed in Walter's investigations was very different from that of Russell, neither was able to announce the cause of the discrepancy, and the problem has remained open up to the present.

Owing to its importance—not only from the dynamical point of view, but also in view of the inferences on the internal constitution of the stars made in this way—it seems worth while to reconsider the problem *de novo*. The fundamental equations of motion from which we shall start in the present paper are the same as those used by Walter, but the method of treatment is quite different and much simpler. As will be seen, our results will justify Russell's formula and disprove the recession of apses. Our main conclusion will be that in close binary systems where the periods of axial rotation of the components coincide with the period of the orbit, the periastron advances—and we shall further prove it even when the librations of the components are taken into consideration, provided only that the amplitudes of the librations are sufficiently small.

2. *The Equations of Motion.*—The equations of motion of our problem in the Cartesian co-ordinates are :

* *A.N.*, 230, 157, 1927 ; 235, 105, 1929.

† *M.N.*, 88, 671, 1928.

‡ *Königsb. Veröff.*, 3, 1933.

§ *Loc. cit.*, p. 14.

$$\left. \begin{aligned} \frac{d^2 x_i}{dt^2} &= k^2 \frac{\partial V_i}{\partial x_i}, \\ \frac{d^2 y_i}{dt^2} &= k^2 \frac{\partial V_i}{\partial y_i}, \\ \frac{d^2 z_i}{dt^2} &= k^2 \frac{\partial V_i}{\partial z_i}, \end{aligned} \right\} \quad (1)$$

where k^2 is the Gaussian constant and V_i the respective potential function (the suffix i referring to each component). Let us suppose the components to be heterogeneous ellipsoids with polytropic density distribution and three unequal axes $a_i > b_i > c_i$, the longest axes lying in the x direction and c_i 's being their axes of rotation. Denote ϵ_i and ϵ_i' their respective meridional and equatorial eccentricities, *i.e.*

$$\begin{aligned} \epsilon_i^2 &= 1 - \frac{c_i^2}{a_i^2}, \\ \epsilon_i'^2 &= 1 - \frac{b_i^2}{a_i^2}. \end{aligned}$$

Let both ϵ_i and ϵ_i' be so small that the effects arising from their fourth (and higher) powers can be neglected; then the potential of each component with respect to the centre of the other takes the form

$$V_i = \frac{m_i}{r} \left\{ 1 + \alpha_i \frac{a_i^2}{r^4} [\epsilon_i^2 (x^2 + y^2 - 2z^2) + \epsilon_i'^2 (x^2 - 2y^2 + z^2)] \right\}, \quad (2)$$

where m_i is the mass of each component, r the distance of their centres and α_i a function of the density condensation which for the polytropic model can very approximately be given by

$$\begin{aligned} \alpha_i &= \frac{1}{2} \left(1 - \frac{1}{\Delta_2(n_i)} \right), \\ \Delta_2(n) &= \frac{5\psi_2(\xi_1)}{3\psi_2(\xi_1) + \xi_1\psi'_2(\xi_1)}, \end{aligned}$$

Δ_2 being expressed in terms of associated Emden functions and tabulated by Chandrasekhar.*

The numerical values of α_i for integral values of the polytropic index n from 0 to 5 are:

n	α
0	0.30000
1	0.17102
2	0.06454
3	0.01404
4	0.00134
5	0.00000

If $\epsilon_i' = 0$, (2) reduces to the well-known potential of oblate spheroids, while if $\epsilon_i = \epsilon_i'$, (2) becomes the potential of a prolate spheroid.

* *M.N.*, 93, 453 and 456 (Table VI), 1933.

Let us change over to polar co-ordinates and transform (1) in the Lagrangian form. The potential energy of the system can clearly be deduced from (2), and in writing out the kinetic energy let us allow for axial rotation of both components. Assume further the plane of the orbit to coincide with the equatorial planes of both ellipsoids. If r denotes the radius vector, v the true anomaly, ϕ_i the angle between a_i and r which in the following will be called the libration, the equations of motion (to the order of accuracy we are working) take the forms *

$$\frac{d^2r}{dt^2} - r \left(\frac{dv}{dt} \right)^2 = -\frac{1}{r^2} - \frac{1}{r^4} \left[K - \sum_{i=1}^2 K_i \sin^2 \phi_i \right], \quad (3)$$

$$r^2 \frac{d^2v}{dt^2} + 2r \frac{dv}{dt} = + \frac{1}{r^3} \sum_{i=1}^2 K_i \sin 2\phi_i, \quad (4)$$

$$\sum_{i=1}^2 T_i \frac{d^2\Phi_i}{dt^2} + \frac{K_i}{r^3} \sin 2\phi_i = 0, \quad (5)$$

where we put

$$\left. \begin{aligned} k^2(m_1 + m_2) &= 1 \\ \Phi_i &= v + \phi_i \end{aligned} \right\} \quad (6)$$

and have used the abbreviations :

$$\left. \begin{aligned} K &= \sum_{i=1}^2 a_i a_i^2 (\epsilon_i^2 + \epsilon_i'^2), \\ K_i &= a_i a_i^2 \epsilon_i'^2, \\ T_i &= \frac{m_1 + m_2}{m_1 m_2} \cdot \frac{2}{3} m_i a_i (a_i^2 + b_i^2). \end{aligned} \right\} \quad (7)$$

3. *The Case of Constrained Rotation.*—Let us first consider the case of purely constrained rotation, *i.e.* assume the periods of axial rotation of both components strictly to agree with the orbital period. If so, we can put $\phi_i = 0$, and the equations of motion take the neat forms

$$\frac{d^2r}{dt^2} - r \left(\frac{dv}{dt} \right)^2 = -\frac{1}{r^2} - \frac{K}{r^4} + \dots \quad (3.1)$$

$$\frac{d}{dt} \left(r^2 \frac{dv}{dt} \right) = 0. \quad (4.1)$$

The second can be integrated as it stands, giving

$$r^2 \frac{dv}{dt} = c_1, \quad (8)$$

and inserting it in (3.1) we get

$$\frac{d^2r}{dt^2} = -\frac{1}{r^2} + \frac{c_1^2}{r^3} - \frac{K}{r^4} + \dots \quad (9)$$

* Cf. Walter, *loc. cit.*, p. 4.

Integrating it we obtain

$$\left(\frac{dr}{dt}\right)^2 = \frac{2}{r} - \frac{c_1^2}{r^2} + \frac{2}{3} \cdot \frac{K}{r^3} + \dots + c_2. \quad (10)$$

Changing now by use of (8) the independent variable, (10) turns into

$$\left(\frac{dr}{dv}\right)^2 = \frac{2}{3} \cdot \frac{K}{c_1^2} r - r^2 + \frac{2}{c_1^2} r^3 + \frac{c_2}{c_1^2} r^4 + \dots, \quad (11)$$

and hence

$$v - c_3 = \int \frac{c_1 dr}{\sqrt{r^2(c_2 r^2 + 2r - c_1^2) + \frac{2}{3}Kr}}. \quad (12)$$

Let us now expand the root under the integral sign in a binomial series, and consistently with our scheme of approximation neglect the terms involving K^2 and higher powers of K . Then (12) can be rewritten as

$$v - c_3 = \int \frac{c_1 dr}{r \sqrt{c_2 r^2 + 2r - c_1^2}} - \frac{K}{3} \int \frac{c_1 dr}{r^2 \sqrt{(c_2 r^2 + 2r - c_1^2)^3}} + \dots \quad (12.1)$$

If $K=0$, *i.e.* if both bodies are either spheres or their density condensations are infinite, (12.1) becomes, as one easily verifies, the well-known expression of the classical two-body problem, and this enables us to determine the values of the constants of integration; we have

$$c_1 = \sqrt{1 - e^2}, \quad c_2 = -a, \quad c_3 = \omega,$$

a , e , ω being the major semi-axis, eccentricity and periastron longitude of the undisturbed orbit respectively. In the following we shall put $a=1$, *i.e.* express the major semi-axes of both components a_i in terms of a as unity.

Now, for $K \neq 0$, (12.1) admits the solution

$$r = \frac{c_1^2}{1 + e \cos(v - \omega')}, \quad (13)$$

where

$$\omega' = \omega - \frac{K}{3} \int \frac{c_1 dr}{r^2 \sqrt{(c_2 r^2 + 2r - c_1^2)^3}} + \dots, \quad (13.1)$$

i.e. the orbits are found again to be conics, but the longitude of periastron ceases to be constant. To determine its variations we have to express the integral from (13.1) in terms of v and perform the integration. For non-circular orbits, in which only we are interested, this can be done by use of (13). Put $v - \omega' = \eta$, and consider ω' for the first approximation to be constant; then substitute (13) in (13.1) and we get

$$\begin{aligned} \int \frac{c_1 dr}{r^2 \sqrt{(c_2 r^2 + 2r - c_1^2)^3}} &= \frac{1}{c_1^4} \int \frac{(1 + e \cos \eta)^3}{e^2 \sin^2 \eta} d\eta \\ &= -\frac{3}{c_1^4} \eta + \text{periodic terms}, \end{aligned} \quad (14)$$

where under "periodic terms" we understand terms multiplied by $\sin nv$ or $\cos nv$ (n being an integer), which clearly cannot contribute to the secular motion of apses in which we are chiefly interested.

Combining (13.1), (14) and (7), we find the total amount of the secular motion of the periastron in our scheme of approximation to be

$$\omega' = \omega + \left\{ \sum_{i=1}^2 \sum_{j=0}^{\infty} \alpha_i a_i^2 (\epsilon_i^2 + \epsilon_i'^2) (1+j) e^{2j} \right\} v. \quad (15)$$

All terms behind the summation signs are positive; the longitude of periastron consequently *advances*, the rate of motion being the higher,

- (i) the greater the relative dimensions of the components,
- (ii) the greater their departures from the spherical shape,
- (iii) the lower their density condensations,

the effect of the orbital eccentricity being comparatively small.

4. *Comparison with Russell's Formula.*—In the case of real binaries the orbital eccentricities are, as a rule, so small that effect of the term involving e can be neglected in a first approximation. Further, as follows from investigations of Chandrasekhar,* the equatorial and meridional ellipticities † are to the first order of approximation given by

$$\left. \begin{aligned} \epsilon_1^2 &= \left(1 + 4 \frac{m_2}{m_1} \right) \Delta_2 \nu_1^3, & \epsilon_1'^2 &= 3 \frac{m_2}{m_1} \Delta_2 \nu_1^3, \\ \epsilon_2^2 &= \left(1 + 4 \frac{m_1}{m_2} \right) \Delta_2 \nu_2^3, & \epsilon_2'^2 &= 3 \frac{m_1}{m_2} \Delta_2 \nu_2^3, \end{aligned} \right\} \quad (16)$$

where ν_i is the radius of a sphere having the same volume as the corresponding ellipsoid. Inserting (16) in (15) and putting, again for the first approximation, $\nu_i = a_i$, we arrive for the ratio of the orbital period (P) to that of the revolution of apses (P') at the expression :

$$\frac{P}{P'} = k_1 a_1^5 \left(1 + 7 \frac{m_2}{m_1} \right) + k_2 a_2^5 \left(1 + 7 \frac{m_1}{m_2} \right), \quad (17)$$

$$k_i = \frac{1}{2} [\Delta_2(n_i) - 1],$$

which agrees exactly with that derived by Russell.‡ Russell's formula is thus found to be right as far as the simplifying conditions introduced above are admissible, *i.e.* as far as the ellipticities of both components are very small. In the case of more considerable ellipticities (17) fails, chiefly because of the breakdown of the supposition $a_i = \nu_i$.§ But as far as the applications are concerned, it has to be pointed out that the determinations of the density condensations in some binary systems made by Russell ‡ and the writer ||

* *M.N.*, 93, 469, 1933.

† The equations (43') of Chandrasekhar's mentioned paper refer to the oblatenesses; we put $\epsilon_i^2 = 2 \times$ the corresponding oblateness.

‡ *M.N.*, 88, 642, 1928.

§ Russell in his paper spoke somewhat ambiguously about the radius of a spheroid, but a little consideration shows that this must refer to its major semi-axis.

|| *A.N.*, 258, 393, 1936.

by use of (17) can be regarded as true—to their order of magnitude at any rate; the results would alter but slightly if we apply the more accurate formula (15) instead of (17).

5. *The Effects of Librations.*—Let us drop now the supposition of the constrained rotation and turn to the effects of librations of the components on their orbits. The equations (3)–(5) present a set which can be reduced, after the elimination of the time, to a system of simultaneous equations with variable coefficients. After having introduced a number of simplifying conditions, Walter* reduced it to a system with constant coefficients which admits a formal integration in the usual way. His results, in a special case treated above, could not, however, be confirmed, and it is a matter of some doubt whether they can be considered as correct.

The simultaneous solution of equations (3)–(5) as they stand would, no doubt, be an extremely difficult task. In order to get an idea about the general nature of the problem let us introduce two conditions which are capable of simplifying the mathematical treatment widely and come at the same time near to our underlying astrophysical problem. These are:

- (i) The orbits are very nearly circles.
- (ii) The amplitudes of librations are small.

Let us suppose the angle of librations to be given by

$$\phi_i = \phi_{0i} \sin (n_i t + \tau_i), \quad (18)$$

where ϕ_{0i} and τ_i are arbitrary constants.† The equations for r and v then become:

$$\frac{d^2 r}{dt^2} - r \left(\frac{dv}{dt} \right)^2 = -\frac{1}{r^2} - \frac{1}{r^4} \left\{ K + \sum_{i=1}^2 K_i \sin^2 [\phi_{0i} \sin (n_i t + \tau_i)] \right\}, \quad (19)$$

$$r^2 \frac{d^2 v}{dt^2} + 2r \left(\frac{dv}{dt} \right) = \frac{1}{r^3} \sum_{i=1}^2 K_i \sin 2 [\phi_{0i} \sin (n_i t + \tau_i)]. \quad (20)$$

Suppose, further, the amplitudes of librations ϕ_{0i} to be of the same order of magnitude as K , *i.e.* quantities the squares of which can be neglected. Then, as the reader easily verifies, the term involving $\sin^2 \phi_i$ in (19) vanishes and (19) becomes identical with (3.1), whilst (20) takes the form

$$\frac{d}{dt} \left(r^2 \frac{dv}{dt} \right) = \frac{2}{r^3} \sum_{i=1}^2 K_i \phi_{0i} \sin (n_i t + \tau_i). \quad (21)$$

Neglecting the quantities of the order $K_i \phi_{0i} e$, we can perform the integration and get

$$r^2 \frac{dv}{dt} = c_1 - 2 \sum_{i=1}^2 \frac{K_i \phi_{0i}}{n_i} \cos (n_i t + \tau_i). \quad (22)$$

Hence

$$r \left(\frac{dv}{dt} \right)^2 = \frac{1}{r^3} \left\{ c_1^2 - 4c_1 \sum_{i=1}^2 \frac{K_i \phi_{0i}}{n_i} \cos (n_i t + \tau_i) + \dots \right\}. \quad (23)$$

* Königsb. Veröff., 3, 4–6, 1933.

† The justification of this formula will be given later (§ 6).

The order of magnitude of the factor $\frac{K_i \phi_{0i}}{n_i}$ deserves our attention. If n_i is not small, $\frac{K_i \phi_{0i}}{n_i}$ would be of the order K_i^2 , *i.e.* negligible, and Kepler's area law would continue to hold good. If, on the other hand, n_i is of the same order as K_i or ϕ_{0i} , $\frac{K_i \phi_{0i}}{n_i}$ would be a quantity of the first order and have to be taken into account.

6. *The Periods of Librations.*—The nature of our problem involves the fact that the n_i 's, *i.e.* periods of librations, are not arbitrary,* but governed by (5). In the case of elliptic orbits their simultaneous solutions with (3) and (4) would confront us with great difficulties. But for circular orbits (which, as already mentioned, come very near to reality) one can easily verify that equations (5) become independent of (3) and (4), and for small amplitudes of librations take the forms

$$\sum_{i=1}^2 \frac{d^2 \phi_i}{dt^2} + 2 \frac{K_i}{T_i} \phi_i = 0, \quad (24)$$

the solutions of which are of the form

$$\phi_i = \phi_{0i} \sin (n_i t + \tau_i),$$

formally anticipated in (18), but the n_i 's are no longer arbitrary, being (with respect to our system of units and to (7))

$$n_i = \left\{ \frac{3\epsilon_i'^2}{2 - \epsilon_i'^2} \cdot \frac{m_i}{m_1 + m_2} \right\}^{\frac{1}{2}}. \quad (25)$$

It turns out that for circular orbits the periods of librations depend

- (i) on the equatorial ellipticity of the respective component,
- (ii) on the mass-ratio of the system,

but do not depend on the internal constitution of the body concerned, the mathematical reason being that the terms involving the density condensation (*i.e.* α_i 's) in K_i and T_i cancel out.† Further, it follows that the periods of libration—compared with the period of revolution—will generally be long. Indeed, n_i will approximately be of the same order as ϵ_i' and, in consequence, the factor $\frac{K_i \phi_{0i}}{n_i}$ in the preceding paragraph has to be taken into account.

The equation which we get by inserting (23) in (19) respectively, (3.1),

* This may easily be understood if we remember that each component has in our analysis three degrees of freedom: two fix the positions in the orbit, while the third determines the orientation of the major semi-axis of each ellipsoid.

† This does *not* contradict directly Walter's results concerning the density condensations in the β Lyrae and W Ursae Majoris systems as far as the librations of higher terms have been used for determinations, but, nevertheless, it may illustrate the extreme difficulty of such a task if we have to deal with nearly circular orbits.

does not admit separation of variables, and its solution would be very difficult. But without entering into detailed calculations we can see that (owing to the neglect of terms involving $\sin^2 \phi_i$ in (19)) no additional terms arise in the secular motion of the apsidal line, and the disturbing term on the right-hand side of (22) will affect the orbital eccentricity which in the conditions under consideration—as generally known—decreases.*

7. *The Case of Larger Librations.*—So far we have supposed the ϕ_i 's to be small quantities the squares of which could be neglected, and we have seen that a breakdown of this condition is apt to imply far-reaching complications. It may be reasonably supposed that in reality the amplitudes of librations will be small and their periods long. Nevertheless, in order to get an idea about the departures from these conditions, it may be worth while to examine an extreme case of librations with the amplitude $\frac{1}{2}\pi$ and period equal to that of the revolution. As one easily verifies, this is the case if the components do not rotate, and the method of variation of elements can be adopted.

Let us turn back to (1) written in a slightly different form :

$$\left. \begin{aligned} \frac{d^2x}{dt^2} + \frac{x}{r^3} &= \frac{\partial R}{\partial x}, \\ \frac{d^2y}{dt^2} + \frac{y}{r^3} &= \frac{\partial R}{\partial y}, \end{aligned} \right\} \quad (1.1)$$

where evidently

$$R = V - \frac{1}{r}.$$

The variation of the longitude of periastron is, as is well known,† given by

$$\begin{aligned} \frac{d\omega}{dt} &= \frac{tg\frac{1}{2}i}{a^2\sqrt{1-e^2}} \frac{\partial R}{\partial i} + \frac{\sqrt{1-e^2}}{a^2e} \frac{\partial R}{\partial e} \\ &= \frac{1}{e} \frac{\partial R}{\partial e}; \end{aligned} \quad (26)$$

if we consider the orbit in a plane ($i=0$), put $a=1$ and neglect quantities of the order e^2 and higher.

Express the disturbing function R in terms of the eccentricity and time, using well-known expansions for the elliptic orbits,‡

$$r = 1 + \frac{e^2}{2} - \sum_{i=1}^{\infty} \frac{2e}{i^2} \cdot \frac{d^2 f_i(i e)}{de} \cos it,$$

$$v = t + 2e \sin t + \frac{5}{4}e^2 \sin 2t + \dots,$$

and we get

$$\frac{\partial R}{\partial e} = - \left(K + \sum_{i=1}^2 K_i \int \sin^2 t dt \right) e + \dots + \text{periodic terms.} \quad (27)$$

* Cf. Walter, *Königsb. Veröff.*, 2, 44, 1931.

† Cf., e.g., Tisserand, *Traité de la Mécanique Céleste*, I, 169 (Paris, 1891).

‡ Cf. again Tisserand, *loc. cit.*, pp. 221, 224.

If $K_i = 0$, i.e. if both bodies are oblate spheroids, (26) yields for the secular motion of periastron in nearly circular orbits,

$$\omega' = \omega + K \cdot t,$$

in agreement with (15). If $K_i \neq 0$, we obtain by the same procedure (and again apart from the periodic terms),

$$\omega' = \omega + [K + \frac{1}{2}(K_1 + K_2)]t,$$

i.e. under our special assumptions $\phi_i = \frac{1}{2}\pi$, $n_i = 1$, $\tau = 0$, new additional terms arise in fact, but they contribute only to the advance of apsides.

8. *Application to Practical Cases and Conclusions.*—In reality, the systems to which the results of the present analysis may be relevant and where it is possible to compare them with observations are, in practice, only eclipsing binaries. In case of Algol-like systems it is nearly always the less massive component which must be appreciably distorted in form,* while the more massive component remains as a rule nearly spherical. We may consequently to a high degree of approximation put $\epsilon_1 = \epsilon_1' = 0$, and (15) assumes a simpler form,

$$\frac{P}{P'} = a_2 a_2^2 (\epsilon_2^2 + \epsilon_2'^2) (1 + 2e + \dots), \quad (15.1)$$

provided that the periods of revolution and of axial rotation of the elliptical component agree—as is for Algol variables likely to be the case. The librations, if present, tend to diminish the orbital eccentricity, but (15.1) continues to hold good as far as the squares of the amplitudes of librations are negligible. If the librations become larger, (15.1) ceases to be true and additional terms may arise, but they will very probably be positive and will consequently make the advance faster.

Observations have now shown in some cases (like Y Cygni), and made it probable in a number of other systems, that the advance of periastron in

* This is a theoretical inference which, so far as I know, has not yet been confirmed by observations. It is of course very difficult to detect it by usual methods. To fix our ideas, let us consider a definite system, say Algol. The visual depths of his minima are 1^m.27 and 0^m.06 respectively, what corresponds to a ratio of the surface brightness of the B and K component $\mathcal{F}_B/\mathcal{F}_K = 36$. It is a matter of simple calculation to show that under such conditions the effect of the elliptical shape of the K component on the visual light-variations of the whole system would be quite insignificant, even if its ellipticity were to be very great. But the ratio $\mathcal{F}_B/\mathcal{F}_K$ depends substantially on the wave-length. In the yellow, as already mentioned, it is nearly equal to 36, and will be still higher in the blue, but will diminish very rapidly with increasing wave-length, and at about $\lambda = 6500$ Å., i.e. in the deep red, the surface brightness of both components will be nearly equalised. Unless the theory is fatally wrong, the light curve of Algol observed in this region of spectrum should show a β Lyræ-like character due to the ellipticity of the K component—the value of which could be determined from the form of the light curve. Similar considerations would apply to all Algol-like systems. Observations of this kind, if carried out for a number of systems, could be extremely important for the study of stellar density condensations from the deformations of their form.

close binaries is a matter of fact; thus the theory and observations qualitatively agree. Quantitatively it provides us with a possibility, from the observed amount of the apsidal motion, to draw some conclusions on the density condensation of each system—if its other elements are known. Russell made the first attempt at determining the degree of density condensation of Y Cygni in this way, and met with the surprising result that its density condensation came out strikingly low. Later, some objections were raised against the correctness of his formula by Walter. The present analysis has plainly justified Russell's ideas, and leaves no room for doubt that the low density condensation arrived at by him in the case of Y Cygni is essentially correct.

In the case of β Lyræ and W Ursæ Majoris systems, where the separations of components are generally smaller and both bodies get appreciably distorted in form, our results are applicable as far as the neglect of the powers of K and K_i can be justified. But it is easy to infer from the general form of our equations that the apsidal line will advance secularly no matter what the form of the components, provided only that the periods of revolution and of axial rotation of both components agree strictly—*i.e.* that Kepler's law of areas holds good. Its breakdown caused by the occurrence of librations with considerable amplitudes would involve such complications of the mathematical treatment of general equations of motion that the outstanding difficulties cannot be overcome at present.

9. *Summary.*—The object of the present paper is to investigate the motion of the apsidal line in close binary systems the components of which are non-homogeneous ellipsoids with three unequal axes and polytropic density distributions. The mathematical analysis in the main part of the paper proceeds by direct integration of the respective equations of motion in the Lagrangian form, and has been carried out up to the second order in the ellipticities of both configurations. The main results are:

- (i) If the orbital period and the periods of axial rotations of both components happen strictly to agree, the longitude of periastron—apart from the periodic changes—advances secularly, the rate of motion being the higher the greater the relative dimensions of both components, the greater their distortions from the spherical form, the lower their density condensations and the greater the orbital eccentricity. The major semi-axis and the eccentricity of the orbit remain secularly constant.
- (ii) But, in general, the nature of our problem requires that in such systems instead of a strictly constrained rotation, long-periodic librations of the components will take place. Then the following is found to be true:—
 - (a) When the amplitudes of the librations are so small that their squares can be neglected, the apsidal line advances by the same amount as in the first case, *i.e.* there are no additional terms to the secular motion of periastron due to small librations, but the orbital eccentricity decreases.

(b) If the librations become larger, the problem has not been solved except for one special case. It is probable that new additional terms would arise for the secular motion of periastron, but these will very probably be positive, *i.e.* will make the advance of periastron faster.

In conclusion, I have great pleasure in expressing my thanks to Professor Sir A. S. Eddington for allowing the very valuable privilege of enjoying the facilities offered by his observatory, where the above work was carried out. My thanks are also due to Dr. R. v. d. R. Woolley, who kindly assisted in reading the manuscript and arranging its final English form.

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ON THE ELLIPTICITY OF CLOSE BINARIES.

W. J. Luyten.

The hope that apsidal motion might offer a clue toward the internal density distribution of the stars has, unfortunately, not been realized, as it seems that the speed of apsidal motion is not accurately predictable at present. The cause of this apsidal motion is presumably the ellipticity of the components, due to rotational and tidal distortion, and it would appear as if a more direct approach to the problem could be made through the observed ellipticities themselves, even though such a test is less delicate than that of the apsidal motion.

Any star whose axis of rotation makes an appreciable angle with the line of sight, and whose equatorial cross-section shows a marked ellipticity due to tidal action of a close companion, will, in the course of a full period of rotation, produce two maxima and two minima of light as a consequence of the varying size of the disk presented. In the vast majority of cases the companion star is so close that eclipses occur, and we have the well-known Algol, Beta Lyræ or W Ursæ Majoris star. However, it is not always possible to separate the eclipse effect from that due to ellipticity in a unique manner as, *e.g.*, in the case of SX Aurigæ, where Oosterhoff's solution with partial eclipses of two spherical bodies and Wyse's solution with partial eclipses of two ellipsoids with a ratio of axes $b/a = 0.88$ give virtually the same representation of the observations.*

For this reason we may first turn to those few variables where the light variations are small and continuous so as to suggest that no eclipses take place, but that the light variations are due entirely to the ellipticity of the rotating primary. Comparison of spectroscopic and photometric observations does indeed show that two maxima and two minima occur during one period of revolution, and that the times of minimum coincide with those of the conjunctions of the components. This explanation has been generally accepted for these "ellipsoidal variables," and it contains nothing inherently improbable since, *e.g.*, a white dwarf of comparatively large mass, revolving close to the surface of an ordinary star, would admirably meet the requirements since it would remain invisible and produce no eclipses but a strong tidal bulge. It may be pointed out, however, that the spectroscopic elements suggest in all cases that the companion star is relatively distant and of small mass, yet the values for the tidal elongation calculated from the observations are no less than that for a large number of binaries where the components are much closer together.

It is obvious that axial rotation and orbital revolution must be synchronous. Let d be the undisturbed diameter of the originally spherical star; let ϵ and ζ represent the rotation and tidal effects, respectively. The

* *B.A.N.*, 7, 107, 1933, and *P.A.S.P.*, 46, 126, 1934.

actual shape of the star may be assumed to correspond to that of an ellipsoid with three unequal axes, the longest of which is the diameter directed at the companion. The lengths of these diameters will then be :

$$\begin{aligned} \text{polar :} & \quad d(1 - 2/3\epsilon - 1/3\zeta), \\ \text{medium :} & \quad d(1 + 1/3\epsilon - 1/3\zeta), \\ \text{longest :} & \quad d(1 + 1/3\epsilon + 2/3\zeta), \end{aligned}$$

which are in the ratio $1 : (1 + \epsilon) : (1 + \epsilon + \zeta)$. In the present synchronous system ϕ , the ratio of the centrifugal force to gravity at the primary's equator is given by $\phi = (m_1 + m_2)/m_1 \cdot (r_1/a)^3$, and similarly the corresponding ratio for the total effective tidal force of the secondary by $\psi = 3m_2/m_1 \cdot (r_1/a)^3$, in which m_1 and m_2 are the masses of the components, r_1 the radius of the primary, and a the semi-axis major of the relative orbit. Hence we shall also have, in first approximation,

$$\zeta/\epsilon = \frac{3m_2}{m_1 + m_2}.$$

The actual values of ϵ will depend upon the internal constitution of the star, and in the case of the planets it has been shown that its extreme values are:

$$\begin{aligned} \epsilon &= 1.25\phi \quad \text{for a Maclaurin model,} \\ \epsilon &= 0.50\phi \quad \text{for a Roche model.} \end{aligned}$$

The observed values for the planets are found to be in good agreement with this, but it might well be thought that the state of matter in a star is sufficiently different to render the above treatment invalid. Chandrasekhar * has investigated the problem thoroughly, assuming the stars to be built up as Emden polytropes, and has found that, to the fifth order in r/a , the tidal and rotational distortions of a double-star component are simply additive. He then derives the following formulæ for the ellipticities of the primary :—

$$\begin{aligned} \epsilon &= \frac{1}{2}(1 + m_2/m_1)(r_1/a)^3 \cdot \Delta_2, \\ \eta = \epsilon + \zeta &= \frac{1}{2}\left(1 + 4\frac{m_2}{m_1}\right)(r_1/a)^3 \cdot \Delta_2, \end{aligned}$$

and hence

$$\zeta/\epsilon = \frac{3m_2}{m_1 + m_2},$$

in which Δ_2 is a function of the polytropic index, decreasing slowly from $\Delta_2 = 2.50$ for a Maclaurin model to $\Delta_2 = 1.00$ for a Roche model. We see that the above formulæ, derived from simple dynamics, still hold exactly for the stars provided these may be assumed to be Emden polytropes in first approximation.

The theories of von Zeipel and Chandrasekhar † indicate that the surface brightness of an elongated star is directly proportional to apparent gravity ; this will result in an increase of the “effective” ellipticity of the stars. With

* *M.N.*, 93, 467, 1933.

† *M.N.*, 93, 573, 1933.

the same notation as above the ratio of maximum to minimum light will be given, in first approximation, by

$$\text{max./min.} = 1 + (1.5 + \alpha)\zeta = 1 + (4.5 + 3\alpha)m_2/(m_1 + m_2) \cdot \epsilon,$$

where $\alpha = 0.2$ for a homogeneous sphere and $\alpha = 0.5$ for a Roche model. A value of 5.5 for the quantity in brackets may be adopted provisionally, corresponding to a density concentration for which $\epsilon = 0.75\phi$. The above effect is tantamount to a darkening toward the centre at conjunction; if there is any appreciable darkening at the limb, or a reflection effect, the "effective" ellipticity becomes somewhat altered again.

The amount of light variation observed will depend upon the orbital inclination of the system; if this is denoted by i we have, finally,

$$\text{max./min.} = 1 + \frac{5.5m_2}{m_1 + m_2} \sin^2 i \cdot \epsilon,$$

or, to the first order of approximation :

$$\epsilon \cdot \frac{11m_2}{m_1 + m_2} \sin^2 i = z = \text{ellipticity constant.}$$

For three ellipsoidal variables, ζ Andromedæ, b_1 Persei and π_5 Orionis, accurate photo-electric light curves have been determined by Stebbins and Huffer,* and spectroscopic orbital elements for the primary are likewise available. While definite information concerning the orbital inclination and the masses and radii of the components is lacking, optimum estimates may be made in the following way. Firstly, we may assume that both components are, in all three cases, normal stars for their spectral class, with normal surface brightness and density. Secondly, it is evident that m_2/m_1 must be large, in order that the tidal effects may become appreciably larger than the rotational, but not so large that the secondary spectrum becomes visible. Thirdly, $\sin i$ must likewise be large, in order that the effect may become observable, but not so large that eclipses occur.

1. ζ Andromedæ. The trigonometric parallax indicates an absolute magnitude of +2.8; three different spectroscopic determinations all make the star brighter than this, and we may therefore assume that its mass is probably not more than about $2\odot$. Its radius might then be estimated at four times that of the Sun, which would yield a density of $0.03\odot$ and imply a rotational velocity of 11 km./sec., at present not verifiable by observation. A favourable situation is further obtained with $m_2 = 0.65\odot$ $i = 83^\circ$, since no secondary spectrum has been observed, and no eclipse.

2. b_1 Persei. The spectroscopic parallax indicates an absolute magnitude of +1.0 slightly brighter than the average. Cannon† originally derived an orbital eccentricity of 0.22 and a mass ratio of 3.7:1. Later observations by Harper‡ and unpublished Yerkes observations show no traces of the secondary component and indicate a virtually circular orbit. Miss Westgate§

* *Ap. J.*, **51**, 218, 1920; *ibid.*, **57**, 1, 1923; *Pub. Washb. Obs.*, **15**, 29, 1928.

† *Pub. D.O.*, **1**, 283, 1911.

‡ *Pub. D.A.O.*, **4**, 309, 1930.

§ *Ap. J.*, **78**, 47, 1933.

has measured the rotational velocity at 50 km./sec., which would correspond to a radius of the order $1.6r_{\odot}$. Owing to the very small value of the mass function it is evident that m_2/m_1 and $\sin i$ cannot both be large. The assumptions $m_1 = 1.5\odot$, $m_2 = 0.4\odot$ (practically Cannon's mass ratio) and $i = 56^\circ$ appear to be the least unfavourable all around.

3. π_5 Orionis. An absolute magnitude of -2.5 is indicated for this B3 star, and hence a mass of about $5\odot$ would be reasonable. Miss Westgate* has estimated the rotational velocity at 40 km./sec., which implies a radius of about $3r_{\odot}$ and a density of 0.2. This latter value seems a little high, and we may perhaps assume $r_1 = 3.5r_{\odot}$, $m_1 = 5\odot$, $m_2 = 1.5\odot$ and $i = 73^\circ$. These would increase the rotational velocity only to 47 km./sec., still in good agreement with the observed value.

The observed facts for these three stars are given in the first eight columns of Table I, the next five columns give the optimum values for the masses, radii and the inclination as calculated on the basis of the above considerations. The last two columns then give the values of ϵ , the pure rotational flattening, as derived from the observed Δm , and the assumed mass ratio and inclination, and of ϕ , the ratio of the centrifugal force to gravity at the equator.

TABLE I

	<i>m</i>	Sp.	<i>M</i>	<i>P</i>	<i>K</i>	<i>V</i> _{rot.}	Δm	$\odot = 1$				<i>i</i> °	ϵ	ϕ
								<i>m</i> ₁	<i>r</i> ₁	<i>m</i> ₂	<i>r</i> ₂			
ζ And	5.1	G8	+2.5	17.77	26	...	0.86	2	4	0.65	0.8	83	0.059	0.0013
<i>b</i> ₁ Per	4.9	A2	+1.0	1.53	39	50	0.52	1.5	1.6	0.4	0.7	56	0.060	0.022
π_5 Ori	3.6	B3	-2.5	3.70	58	40	0.53	5	3.5	1.5	1.6	73	0.043	0.0058

For the average star we might perhaps expect values in the neighbourhood of $\epsilon = 0.6\phi$, and the results derived above appear to be in violent disagreement with theory, the ratio ϵ/ϕ attaining the values 45, 2.7 and 7.4 respectively.

Before commenting upon this discrepancy there may first be discussed other observational determinations of the rotational velocities which came to hand after the above calculations had been finished, viz. those by Colacevich.† Using Lick spectrograms, he derives a rotational velocity of 105 km./sec. for *b*₁ Persei and of 130 km./sec. for π_5 Orionis; hence the radii of these stars would become 2.1 and 3.2 times larger. Other things being equal, ϕ is proportional to r^3 , but in addition to this *i* has to be reduced to about 54° for π_5 Orionis in order to avoid eclipse (unless the companion is a white dwarf). A rough calculation then indicates that for both stars ϕ becomes of the order of 0.2. Similarly, we might increase the rotational velocity of ζ Andromedæ to 50–60 km./sec., which would necessitate a radius of 20 times that of the Sun, and which would then yield $\phi = 0.2$ with $\epsilon = 0.13$.

Numerically, therefore, it is always possible to attain reasonable agreement

* *Ap. J.*, 77, 143, 1933.

† *Pub. Arcetri*, fasc. 55, 21, 1937.

between the values of ϕ and ϵ by increasing the dimensions of the stars. However, with the known data concerning the absolute magnitudes, and hence also concerning the masses, such increases would lead to the following values for the densities, surface brightnesses and temperatures :—

		$V_{\text{rot.}}$	$\rho/\rho_{\odot}$	$\mathcal{F}/\mathcal{F}_{\odot}$	T
ζ And	G8	55	0.00025	0.02	2100° K
b_1 Per	A2	105	0.042	0.75	5400
π_5 Ori	B3	130	0.006	6.5	9200

These values appear to be entirely incompatible with our present ideas and, so long as we retain the assumption of synchronism—and this is a *sine qua non* for ellipsoidal variation—we are forced to conclude that Colacevich's values for the rotational velocities of these stars are unacceptable. It does not necessarily follow that the values used in the present paper must be adopted, but it does appear to the writer that no reasonable increase in them could bring agreement between ϕ and ϵ , especially in the case of ζ Andromedæ.

The explanation of the observed light variation as due to ellipticity of figure is thus seen to lead to grave difficulties. Yet the continuous character of these light variations, coupled with the fact that the spectroscopic data do not permit the systems to consist of stars nearly enough in contact, rules out their explanation through eclipses. The spectroscopic values of $a \sin i$, of the same order of magnitude as the probable radii of the stars, combined with the very small range in magnitude, as well as the occurrence of *two* maxima and *two* minima during one period, likewise rules out Cepheid variation. The synchronization of the minima with conjunctions points inescapably toward a connection between the light variation and the secondary, yet the results of this action of the secondary appear to be many times greater, relatively, than in the ordinary eclipsing system.

We may now turn our attention to those eclipsing binaries where estimates of the ellipticities have been made, and confine ourselves chiefly to three classes of objects, viz. :

(1) Stars of the W Ursæ Majoris and Beta Lyræ type where the two components, nearly equal in size, are revolving nearly in contact. The calculated ellipticities are large, but, as the stars are faint, spectroscopic data are available in only a few cases.

(2) Algol stars with good spectroscopic orbits and light curves. For many of these two spectra are visible, and the mass ratio is known directly, while photo-electric light curves are available for several.

(3) Eclipsing binaries of long period, generally belonging to the late-type giants, and whose calculated ellipticities appear suspicious *a priori*.

Table II gives a summary of the necessary data for these stars. The first four columns give the name, the spectral class, the period and the relative radius of the primary, column 5 the mass ratio, italicized numbers indicating estimates based upon the observed difference in magnitude. The sixth column shows the ratio of the axes of the equatorial cross-section, b/a , as published by the computer of the photometric elements. The orbital inclination and the effects of reflection and darkening at the limb have already

TABLE II

Star	Sp.	P d	r/a	m_2/m_1	b/a	ϕ	ϵ	ϵ/ϕ
RV CVn	F8	0.27	.35	.8	.81	.091	.078	.86
VW Cep	G5	0.28	...	.8	...	.161	.109	.68
TZ Boo	...	0.30	.42	.9	.8	.144	.079	.55
SW Lac	G2	0.32	.43	.8	.81	.17	.082	.48
AB And	...	0.33	.42	.9	.74	.14	.10	.71
W UMa	F8	0.33	.43	.72	.79	.14	.09	.64
RT Scl	A5	0.51	.29	.9	.78	.162	.087	.54
S Ant	A8	0.65	.50	.56	.75	.185	.127	.69
GO Cyg	B9	0.72	.38	.84	.86	.106	.057	.54
SX Aur	A3	1.21	.42	.6	.88	.113	.058	.51
TT Aur	B3	1.33	.39	.80	.84	.106	.066	.62
V Pup	B1	1.45	.46	.8	.83	.17	.07	.4
VV Ori	B2	1.48	.40	.3	.96	.076	.036	.48
AH Cep	B0	1.77	.35	.86	.92	.080	.032	.40
β Lyr	B8	12.92	.22	.8	.76	.2	.1	.5
W Cru	G0	198.5	.5	.7	.8	.21	.09	.42
σ Aql	B3	1.95	.24	.83	.96	.026	.016	.62
u Her	B3	2.05	.32	.38	.915	.068	.053	.78
β Aur	A0	3.96	.16	1.00	.99	.008	.004	.5
X Tri	A3	0.97	.32	.6	.919	.057	.038	.67
RZ Cas	A2	1.20	.25	.4	.975	.03	.02	.67
U Oph	B5	1.68	.25	.88	.94	.031	.024	.77
RX Her	A0	1.78	.21	.89	.97	.016	.012	.75
TV Cas	A0	1.81	.28	.59	.95	.033	.025	.76
δ Lib	A0	2.33	.31	.5	.95	.047	.026	.55
Z Vul	B3	2.45	.30	.45	.91	.052	.064	1.24
β Per	B8	2.87	.21	.25	.987	.011	.008	.73
λ Tau	B3	3.95	.30	.4	.95	.038	.031	.82
RS Vul	B8	4.48	.28	.31	.98	.027	.015	.55
RT Lac	G9	5.07	.27	.54	.96	.041	.021	.51
RZ Cnc	K0	21.6	.24	.75	.86	.100	.060	.60
RX Cas	G2	32.3	.28	.9	.85	.041	.060	1.5
SX Cas	A6	36.6	.12	.9	.85	.017	.060	3.5
RU Cen	G6	64.7	.29	.75	.75	.112	.103	.9
ζ And	G8	17.77	.10	.32	.920	.001	.059	45.
b_1 Per	A2	1.53	.24	.27	.930	.022	.060	2.7
π_5 Ori	B3	3.70	.19	.30	.946	.006	.043	7.4

been taken into account in the derivation of b/a . Considering Chandrasekhar's relation between surface brightness and apparent gravity, these values of b/a correspond to the "effective" ellipticity; the actual ellipticity, here called ζ , will be less. The ratio ϕ of the centrifugal force to gravity at the equator is given in column 7, where, it should be added, an allowance has been made for the contribution of the secondary. If, e.g., the two stars are nearly equal in light but very different in size, the ellipticities may well be different, and the observed value is a mean of those of the components.

The value of ϕ given has been calculated in such a way as to correspond most closely to that of ϵ given in the eighth column. This last quantity, *i.e.* the pure rotational flattening the star would have if it rotated with its present speed, but in the absence of all tidal action, is related to b/a , according to the formulæ derived above, by

$$\epsilon = (m_1 + m_2)/5.5m_2 \cdot (1 - b/a).$$

The ratio ϵ/ϕ is given in column 9; for comparison the data for the three ellipsoidal variables discussed first are added at the bottom. The star W Crucis, in spite of its long period, should probably be classed with the W Ursæ Majoris group. The O giants AO Cassiopeiæ and 29 = VW Canis Majoris, as well as the M giant VV Cephei, have not been added, since for these stars the photometric constants are as yet too uncertain. For Y Cygni it now appears certain that the stars rotate faster than they revolve, hence there should be no appreciable tidal effect, which is borne out by the observations from which no ellipticity is evident. [This state of affairs was predicted by Walter * from the observed *advance* of periastron. While his theory for the regression of the line of apsides for synchronized rotation and revolution appears to be definitely contradicted by the observations, this particular prediction has been verified.] For VW Cephei several photometric solutions are available, and the values given for ϕ and ϵ are means derived from two solutions given by Walter,† two solutions given by Dugan‡ and a solution made by myself from van Gent's observations.

Considering the difficulty of separating eclipse and ellipticity effects for the stars of the first group, one might well expect the results for these stars to be the least reliable, while those for stars with small values of r/a and consequent narrow eclipses should be more accurate. A glance at Table II suggests no great differences in relative reliability. The mean value of ϵ/ϕ for the first group is 0.57, in good agreement with what theory has led us to expect, and indicating that the central density concentration for these stars is somewhat greater than for the planet Saturn. For the stars of the second group we find $\epsilon/\phi = 0.71$, corresponding to a central density concentration intermediate between those of Jupiter and Saturn.

Whether the difference is significant is hard to say, as everything depends upon the validity of the formula used for the calculation of ϵ from the observed value of b/a , and the derivation of the latter quantity from the observations. The constant 5.5 used in this formula depends again on the assumed density concentration; if this is to be adjusted so as to correspond to the final value found for ϵ/ϕ , it would lower the mean value for the first group to about 0.55 and that for the second group to 0.70.

There are indications of some systematic effects in the calculation of the ellipticities from observations of eclipses for stars of very low density: the smallest values for ϵ/ϕ , smaller even than the theoretical minimum, occur for the three stars of earliest spectral class and for the yellow giant W Crucis.

* *Kl. Veröff. Königsberg*, No. 3, 1933.

† *Ibid.*, No. 7, 1936.

‡ *Princeton Contr.*, No. 13, 1933.

Likewise, such data as are available for AO Cassiopeiæ and VW Canis Majoris would lead to values of ϵ/ϕ for these O giants of only 0.1 and 0.3, respectively.

The results obtained are not at variance with our expectations, but beyond this we cannot go. Considering all the uncertainties involved, it would be impossible to say what the real internal density concentration of the stars is; a small change in ϵ/ϕ when this is near 0.50 has an enormous influence on the resulting central density, but it is evident that the direct observations of ellipticity are not delicate enough for this purpose. In principle the effects produced as apsidal motion are more suitable, since here the observed advance of periastron is proportional to $\epsilon/\phi - 0.5$; but, as stated at the beginning, experience has shown that the dependence of apsidal motion on the ellipticities of the stars is not as simple as had been supposed.

Throughout the present paper, as well as in the derivations of ϵ and ϕ by Chandrasekhar, it has been assumed that the stars rotate as rigid bodies with constant ω . If this is not so, we may distinguish two cases, viz. (a) ω increases toward the centre, and (b) ω decreases toward the centre. If two stars of the same size and similar internal density concentration rotate with the same surface velocity, but if in star A ω increases toward the centre while B rotates as a rigid body, then the observed polar flattening of A will be greater than that of B. What is observed, however, in synchronous eclipsing binaries of known masses and dimensions is the *tidal* elongation, and hence it would seem that, since this is not altered by a change in ω toward the centre, our conclusions are independent of the assumption of constant ω .

In the case of apsidal motion the situation is different: here the observed rate of change of ω is proportional to $\epsilon - \phi/2$, which would be greater in star A above than in star B. Since nearly all known cases of apsidal motion are *slower* than would be expected from a reasonable density concentration, this would point in the direction of a *decrease* in ω towards the centre. This, in turn, would suggest that the surface rotation has been imposed upon the more slowly rotating interior by the orbital motion of the companion, and hence would suggest the origin of the binary not through fission, but through capture.

Summary.—Owing to the failure of theoretical predictions of apsidal motion in close binaries to conform with the facts of observation, an attempt is made to estimate the degree of central density condensation in these stars from the observed ellipticities. For the three well-known ellipsoidal variables the ellipticities calculated from the light variations are found to be considerably greater than those permitted by theory. For a number of eclipsing variables the values found for ϵ/ϕ correspond to a central density condensation about the same as for the planet Saturn.

Leiden :
1938 March 1.

ON THE ESTIMATION OF SPECTRAL TYPE AND LUMINOSITY IN B-TYPE STARS.

E. G. Williams and D. L. Edwards.

Introduction.—The degree of precision of classification of the stars of type B has never equalled that of the later spectral types. This is largely due to the stars themselves; as a group they offer wide vagaries of line character as well as intensity anomalies among lines of the different elements, and present more cases of individual peculiarity than any other of the main spectral divisions. Apart from this, there is irregularity in passing from B₀ to B₉ because the Harvard system, on which some revised classifications such as that emanating from Victoria are also based, neglects entirely subdivisions B₄, B₆ and B₇.

The problem of obtaining a system which shall bring together those stars with most nearly the same *average* characteristics has been fully discussed by Williams,* who elaborated a method which bears more relation to a physical classification than that provided by the Henry Draper criteria, and at the same time distributes the stars more uniformly throughout the subdivisions.

For determining the spectral sub-type, line ratios exclusively are used, while absolute intensities are reserved for subsequent sorting of the stars into luminosity groups. This division of method provides criteria for type which are insensitive to changes in absolute magnitude, and *vice versa*, and the scheme is thus similar in effect to that recently outlined by Morgan † for types A to K.

As applied originally, the method depended on photometrically measured line intensities. The present contribution is the result of an attempt to devise a parallel scheme, made as simple as possible, for application to unstandardised spectra by mere visual inspection, since observations of this sort must continue for some time to be the basis of future classifications—especially of large numbers of stars for statistical purposes. In addition to providing a method of type determination for general use which is more rational than the earlier systems, it explains a new and simple method of spectroscopic parallax determination for B-type stars which it is hoped to extend to more plentiful material at a later date.

Material.—In the paper by Williams referred to above, “measured” spectral types were derived for about 80 O- and B-type stars for which standardised slit spectra were obtained at Mount Wilson. Objective-prism spectra of 45 of these objects have been selected from the Norman Lockyer Observatory collection, and the line ratios and intensities studied for these with respect to the measured types. In this manner the regularity aimed at

* *Ap. J.*, **83**, 305, 1936.

† *Ap. J.*, **85**, 380, 1937.

in the original derivation of type for these stars was transferred to the visual-estimates scheme.

The remaining stars in Williams's list were either fainter objects not yet photographed at the Norman Lockyer Observatory, or stars unsuited to the present investigation (such as γ Cas, or double-star components which are unseparable in objective-prism spectra). All the spectra had been taken with the 12-inch McClean telescope, giving a dispersion ranging from 47 Å./mm. at $H\beta$ to 23 Å./mm. at $H\epsilon$.

The material was treated independently by both authors, one of whom was relatively unfamiliar with the stars employed. For 40 of these stars two or more negatives were available. The negatives varied considerably in quality, while the stars themselves included many with peculiarities of different kinds, so that the whole collection formed a severe test of the practical value of the method.

Intensity Estimation.—One of the advantages of using line ratios for type determination is that each spectrum can be studied as a unit; the lines which are to be used have had the same photographic treatment and can be examined together under the eyepiece. If, on a second spectrogram of the same star, the intensities are all estimated on a different scale, the final result varies but little. We adopted a numerical scale of intensities roughly represented as follows :—

Barely discernable	1	Moderately strong	5
Faint	2	Strong	7
Quite distinct	3	Very strong	10

with intermediate values when necessary. The actual values of the estimated intensities might vary slightly from plate to plate, but care was taken to preserve the above-mentioned essential quality, namely, that the scale should be constant for all the lines on any individual plate.

The ratios defining type were made to depend on lines of as many different elements as possible. For practical purposes we were limited to lines of seven elements as follows :—

<i>HeI</i> 4713, 4144, 4121, 4009.	<i>OII</i> 4070, 4072, 4076, 4415,
<i>HeII</i> 4200, 4542, 4686.	4417, 4642.
<i>CII</i> 3920, 4267.	<i>MgII</i> 4481.
<i>CIII</i> 4069, 4187.	<i>SiII</i> 4128, 4131.
<i>NII</i> 3995, 4447, 4631.	<i>SiIII</i> 4553, 4568.
<i>NIII</i> 4097, 4379.	<i>SiIV</i> 4089, 4116.
	<i>FeII</i> 4233, 4508, 4583.

Owing chiefly to their large intensities relative to other lines, the hydrogen and stronger helium lines were omitted from these estimates, being reserved for the luminosity criteria mentioned later.

In order to facilitate identification a "line indicator" was used. This, reduced photographically from a drawing, was a clear glass plate on which were printed the positions in the spectrum of the lines listed above together with those of hydrogen and helium. The plate was fixed on a support in

front of an illuminated diffusing screen and the spectrum to be examined was placed on top, film downwards, and moved till coincidence was secured for the hydrogen lines. Estimates were made with the aid of a watch-maker's eyepiece adjusted at a fixed distance over the negative. This arrangement was found to be highly successful in dealing with faint lines which would otherwise have been completely overlooked.

Type-defining Ratios.—Four ratios were used, and may be designated as follows :—

$$A = \frac{HeI}{MgII, SiII, FeII};$$
$$C = \frac{CII, NII, OII, SiIII}{CIII, NIII, SiIV};$$

$$B = \frac{MgII, SiII, FeII}{CII, NII, OII, SiIII, SiIV};$$
$$D = \frac{HeII}{HeI}.$$

In each case the intensities of all observed lines (in the list given above) of the atoms included in the numerator were summed and divided by the similar sum for those in the denominator. This gave the required numerical ratio. The lines of the atoms grouped together are those which change in strength in a similar manner over the spectral range for which they are used. The calcium [K] line might be added to the denominator of ratio *A* in working with spectra which are consistently well exposed in the ultra-violet.

The adopted values of the four ratios for each sub-type appear in Table I. In deriving a mean type from curves representing each ratio it is important to note the part of each curve used and to give greater weight to sub-types from ratios falling on the middle part than to those falling towards the ends. It will be noticed that the ratio curves differ somewhat for the two observers.

TABLE I
Mean Values of Ratios

Type	A		B		C		D	
	W.	E.	W.	E.	W.	E.	W.	E.
O6							5.0	1.8
O7					...	0.8	2.9	1.4
O8					0.0	1.4	1.3	1.0
O9					0.1	2.2	0.6	0.7
B0					0.6	3.4	0.1	0.4
B1	10.5	8.0	0.0	0.2	3.6	5.2	0.0	0.2
B2	6.5	5.2	0.2	0.4	9.5	7.4		
B3	4.1	4.0	0.4	0.6	13	9		
B4	2.8	2.9	0.7	0.8				
B5	2.2	2.0	1.1	1.1				
B6	1.7	1.2	2.5	1.6				
B7	1.1	0.7	5.2	2.3				
B8	0.5	0.5	11	4				
B9	0.3	0.3						

This, however, did not seriously affect the final results, and simply serves to exhibit the independence of the two observers.

The classifications for each star examined are shown in the first eight columns of Table II. Columns 3 to 7 contain the sub-types from each ratio and the weighted mean by Edwards; column 8 this mean by Williams, and column 9 the "measured" type as found by Williams from spectrophotometric observations of similar ratios. The mean difference between the estimates of the two observers is 0.55 sub-type, while that from the "measured" type is 0.70 and 0.73 for Edwards and Williams respectively. These large residuals are due chiefly to diffuse-line stars; when all *n* and *nn* objects are omitted the mean residuals become 0.4 and 0.6.

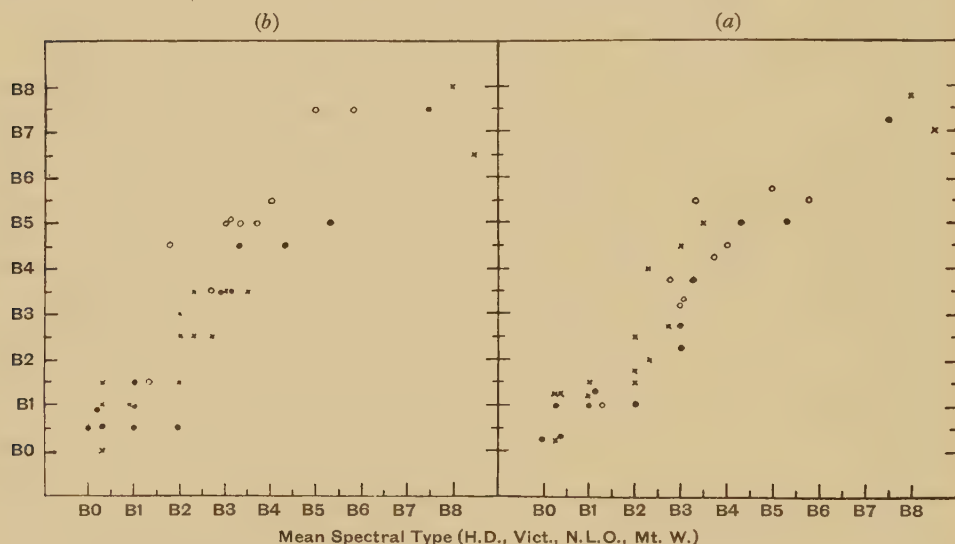


FIG. 1.—Relation between mean spectral type and (a) present estimated type, (b) Williams's "measured" type (Mt. W.).

Fig. 1 shows (a) the relation between the present estimated type (mean of the two observers) and the mean spectral type from the H.D., Victoria, Norman Lockyer, and Mt. Wilson (Adams and Joy) classifications; and (b) that between Williams's "measured" type and the latter mean. It can be clearly seen that the diffuse-line stars (circles) are found to be about 1.5 later in sub-type when the classification depends on *measured*-line intensities. In visual work our greatest trouble comes from these B_n stars some of which show hardly a line beyond those of hydrogen and helium. For such stars of types later than B₂ assignment must depend primarily on the *H/He* ratio; their misclassification from low-dispersion plates is all too probable.

Luminosity Grouping.—The absolute magnitude effects on B-type spectra have been fully investigated by Williams* and Rudnick.† In addition to the well-known enhancement of hydrogen lines for the low-luminosity

* *Loc. cit.*

† *Ap. J.*, 83, 439, 1936.

TABLE II

Name, or H.D. No.	H.D. Type	Type, from Ratios (D. L. E.)				Mean Type		"Measured" Type	Abs. Mags.		
		A	B	C	D	D. L. E.	E. G. W.		D. L. E.	E. G. W.	"Measured"
γ Peg	B2	2.0	2.0	2.5	...	2.0	2.0	2.5	-3.5	-3.3	-3.1
ζ Cas	B3	2.0	2.5	2.0	...	2.5	3.0	2.5	-3.5	-2.3	-3.1
o Cas	B2	5.0	6.0	...	...	5.5	5.5	5.0	-1.5	-0.2	-1.9
e Cas	B3	4.5	4.5	...	...	4.5	5.5	4.5	-3.3	-2.9	-2.1
21291	B9p	8.5	6.5	...	...	7.0	7.0	6.5	-5.2	-6.0	-5.5
δ Per	B5	4.8	6.0	...	...	5.5	5.5	7.5	-1.1	-1.8	-1.1
ζ Per	B1	...	0.5	1.0	1.5	1.0	2.0	1.0	-5.7	-5.5	-5.5
5 Per	Oe5	...	...	O7	O7	O7	O7	O8	-5.0	-4.7	-4.7
9 Cam	Bo	...	...	O8	O8.5	O8	O9	O9	-5.5	-5.1	-5.6
11 Cam	B3p	3.0	4.0	...	...	3.5	3.0	5.0	-1.5	-1.8	-1.9
η Aur	B3	3.0	4.5	...	...	4.0	3.5	4.5	-1.1	-1.7	-1.1
β Ori	B8p	8.2	8.3	...	...	8.0	7.5	8.0	-6.1	-6.1	-5.5
β Tau	B8	7.0	6.0	...	...	6.5	8.0	7.5	-2.9	-3.1	-1.1
χ Aur	B1	4.9	4.0	...	...	4.0	4.0	3.5	-5.3	-5.3	-5.7
e Ori	Bo	...	0.5	O9	0.0	0.0	0.5	0.0	-5.6	-4.9	-5.8
k Ori	Bo	...	...	O9.5	0.0	0.0	0.5	0.5	-5.2	-5.2	-5.3
139 Tau	B2	...	1.0	0.5	0.8	0.5	1.5	0.5	-4.4	-4.4	-4.3
χ ² Ori	B2p	...	0.6	1.5	1.5	1.5	1.5	1.5	-5.5	-6.2	-5.9
β CMa	B1	...	0.9	1.0	1.0	1.0	1.5	1.0	-3.9	-4.5	-4.0
η Hya	B3	4.0	3.5	...	...	4.0	4.5	5.0	-2.0	-2.2	-1.9
η Leo	Aop	9.0	8.0	...	...	8.5	8.0	8.0	-4.4	-4.5	...
γ Leo	B8	...	...	...	...	...	8.0	8.0	-0.7	-1.3	-1.0
23 Sex	B3	3.5	4.5	...	...	4.0	3.5	3.5	-3.0	-3.4	-2.5
o Leo	Bop	...	1.0	1.3	1.5	1.0	1.5	1.0	-5.3	-5.5	-5.5
k Dra	B5p	4.7	4.8	...	...	5.0	6.5	7.5	-1.4	-2.2	-1.2
η UMa	B3	4.4	4.1	...	...	4.0	2.5	5.0	-2.3	-2.7	-0.9
r Her	B5	5.1	5.5	...	...	5.5	4.5	5.0	-2.1	-2.7	-1.9
χ Oph	B3p	2.0	2.9	1.5	...	2.5	1.0	...	-2.8	-1.7	-2.8
5 Oph	Bo	...	1.0	0.5	O9	0.0	O9.5	...	-3.1	-2.8	-4.1
149881	B2	...	0.5	1.0	0.5	0.5	1.5	0.5	-4.3	-4.4	-3.7
Her	B3	4.0	5.0	...	...	4.5	4.5	3.5	-2.7	-2.7	-2.5
88 Her	B8	7.6	7.0	...	...	7.5	8.5	8.5	-2.0	-1.3	-1.0
57 Oph	B5p	4.7	4.8	...	...	4.5	5.5	3.5	-5.5	-5.4	-5.6
2 Vul	B3	3.7	4.2	...	...	4.0	5.0	5.5	-2.0	-2.1	-0.8
90603	Bo	...	1.1	1.0	1.0	1.0	1.5	1.5	-5.6	-6.0	-5.1
93322	B2p	...	...	O9	O8.5	O9	O9.5	O8	-4.5	-4.9	-5.0
5 Cyg	B2	3.5	2.0	...	...	2.5	2.5	3.0	-5.3	-5.7	-5.7
9 Cyg	Bo	...	0.5	O8	0.5	0.0	0.5	0.5	-4.8	-4.9	-3.9
3 Cep	B1	...	1.0	0.5	1.3	1.0	1.5	1.5	-4.0	-3.4	-3.6
Cep	B2p	2.0	1.5	1.0	...	1.5	2.0	2.5	-5.5	-4.9	-5.5
08947	B3	3.5	2.5	...	...	3.0	2.5	3.5	-1.5	-1.3	-2.8
Cep	Od	...	...	O8	O6	O7	O6	O6	-4.7	-4.7	-5.1
Aqr	B1p	...	2.0	0.0	1.0	1.0	1.0	1.5	-3.7	-4.0	-3.4
Lac	B3	3.0	2.0	2.5	...	2.5	2.0	3.5	-3.1	-2.7	-3.3
24151	Bo	...	0.7	1.0	0.8	1.0	1.0	1.0	-5.2	-5.5	-3.8

stars, an increase in total intensity of the helium lines with decreasing brightness was clearly brought out. Both effects are pronounced and regular enough for luminosity grouping. The hydrogen effect persists throughout type B, though it is relatively small at B₀; for helium the types later than B₅ are insensitive.

It is impossible to make useful eye estimates of total intensity for broad hydrogen and helium lines. As an instance of this we may cite the case of 67 Ophiuchi (B3.5s, giant) and ι Herculis (B3.5ss, dwarf). The difference in luminosity is probably about three magnitudes. When viewed together under an eyepiece it was found barely possible to differentiate between the

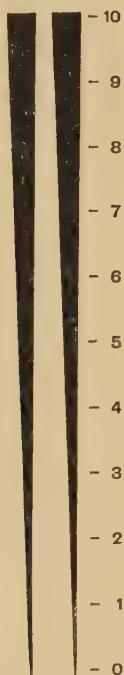


FIG. 2.—Photographic wedges used for line-width measurement.

two sets of hydrogen lines. Yet these lines are more than twice as intense in the dwarf. A similar case is offered by 9 Cephei (B2.5s, giant) and γ Pegasi (B2.5ss, dwarf) with roughly the same luminosity ratio. The difficulty arises because the strong core of the line upsets the judgment of the eye as to the extent of the wings.

The following simple device not only removes the cause of the difficulty (*i.e.* the line core), but enables a numerical estimate of total line-width to be made. Two equal small-angle opaque wedges were made by photographically copying a line drawing and printing on an otherwise clear process plate. The wedges were made to lie parallel, with their lengths vertical, and a graduated scale from 0 to 10 units added at the side (fig. 2). The size of each wedge was such that one unit = 0.15 mm., corresponding to about 5 Å. at $H\gamma$ on the spectra measured. Tenths of a unit were judged by estimation. This plate was held fixed in a viewing-frame and the plate

to be measured placed over it, film down, with the spectrum horizontal. The absorption line to be examined was then brought directly under the apex of the left-hand wedge, and the spectrum raised till no sign of the absorption could be seen—*i.e.* the part of the wedge at the level of the spectrum was broad enough to mask all parts of the line visible to the eye.

The second wedge is necessary as a check against optical illusion. Even if no absorption exists, the parts of the continuous spectrum adjacent to the black wedge appear to be brighter than the rest. With two wedges one can seek the position where the two appearances just cease to match. In practice it was also found possible to begin at the thick end of the wedge, with the line completely hidden, and to take a reading of wedge position where absorption is first certainly seen, and this method was actually used by one observer while the other adopted the first-mentioned procedure.

On a scale ranging from 0 to 7, the difference between the mean hydrogen readings for ι Herculis and 67 Ophiuchi was 2.1 units.

A smaller pair of wedges was used for the helium lines. In general, four hydrogen and four helium lines were measured, and in each case the mean reading (or the adjusted mean when less than four lines were measurable) was taken as the measure of wing width.

It was found necessary to employ an approximately constant source of illumination for these observations, since any great change (such as a transition from bright Sun to cloudy sky if daylight were used) affected the estimated line widths. Daylight could be used if reasonably constant for the whole series of measures, but an artificial source (a "daylight" electric bulb with diffusing screen) of constant intensity was found to give more reliable results. The plate was enclosed in such a way as to exclude all other (possibly variable) sources of illumination. Under suitable conditions considerable accuracy could be obtained and observations made at different times agreed within two- or three-tenths of a unit. Differences between differently exposed plates of the same star were rather more important, but rarely caused variations of more than about half a unit even in extreme cases.

The wing width correlates reasonably well with the ratio (total absorption)/(line depth) derived from the photometric intensities, which measure of line width turns out to be a good index of luminosity. Certainly for hydrogen it is a better one, in the earlier sub-types at least, than the total absorption alone. A quantity proportional to the total absorption was used by Opik and Olmsted* in their spectroscopic evaluation of absolute magnitudes.

In deriving luminosities from wedge-estimated widths we have used only two curves, one for hydrogen and one for helium, derived through a plot of line widths against the mean absolute magnitudes for the respective stars found by Williams (*loc. cit.*). In view of the inherent uncertainties further refinement is unnecessary. The reduction curve for hydrogen, shown in fig. 3, is the mean for the two observers. The individual curves agree extremely well, and on the scale of this diagram are only just distinguishable from each other. This curve is applicable to all sub-types from Bo to B8.

* *Harv. Circ.* No. 380, 1932.

That for helium is restricted to sub-types Bo to B₄; it is of similar form to the hydrogen curve, but based on measures with a narrower wedge and consequently using a different unit of line width. When available, helium values were usually given equal weight with hydrogen.

Table II shows, in columns 10 and 11, the absolute magnitudes derived by both authors using independent sets of curves. Williams's "measured" values are given in the last column for comparison. The average difference (regardless of sign) between the absolute magnitudes derived by the two observers is 0.4 mag.; that between Williams's "measured" values and the

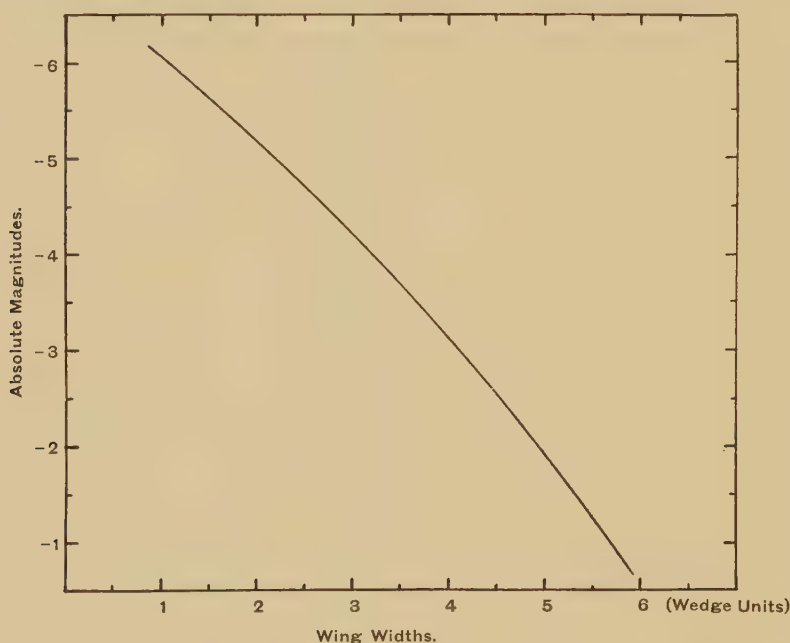


FIG. 3.—Relation between absolute magnitude and wing width for hydrogen.

mean of the present ones is 0.5 mag.; while the mean difference between results obtained from hydrogen lines and those from helium is 0.3 mag.

Fig. 4 (a) shows the relation between the absolute magnitudes (mean) obtained by the present method, and Williams's original values on which they are based; the agreement is good, especially if the diffuse-line stars (indicated by small circles) are omitted. In fig. 4 (b) the relation is shown between the present mean absolute magnitudes and the mean of all previous spectroscopic values by earlier methods. In addition to the greater scatter, the large scale-difference between the two systems is obvious (as previously noted by Williams).

The results of this method for deriving luminosities should give spectroscopic parallaxes for B-type stars which are a great improvement on the earlier determinations, since they are based on the more recent results of spectrophotometric investigation and on more reliable distances for the stars used as a foundation for the method.

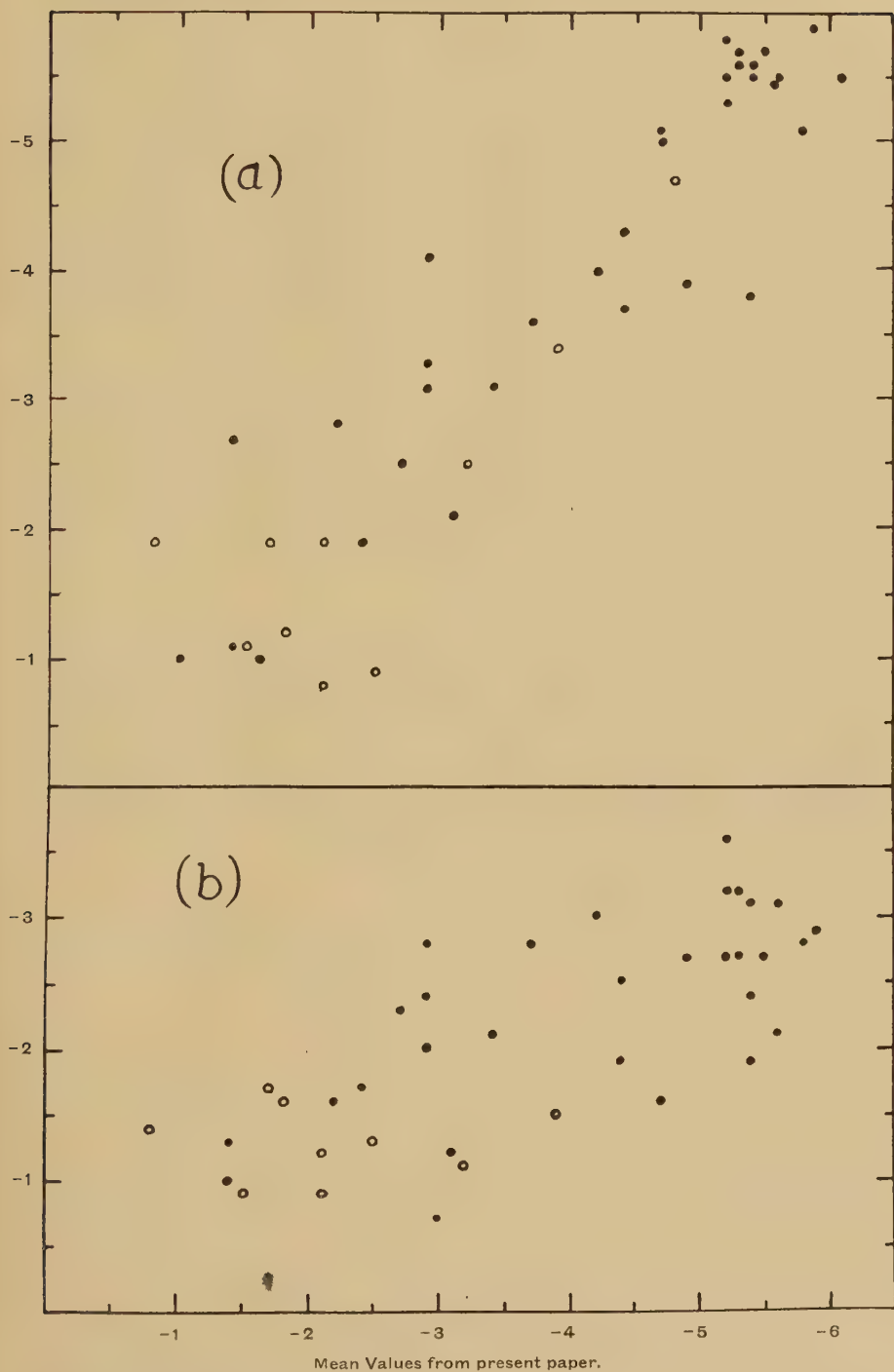


FIG. 4.—Relation between absolute magnitudes from the present paper and (a) Williams's "measured" values (*Mt. W.*), (b) mean of previous spectroscopic values.

The time involved in the necessary measurements is relatively short. The line-intensity estimates for type take about five minutes to make ; so does each set of width measures. Hence about three stars can be fully measured and reduced per hour.

At the Norman Lockyer Observatory spectra of about 700 B-type stars (mostly brighter than mag. 6.5) are available, taken with the same instrument and the same dispersion as the 45 stars investigated in the present paper. In addition, the list may be extended to the remaining stars of these magnitudes, and even (if cases of special interest warrant the increased observing time required) to stars of magnitude 7.5. It is hoped that the present methods may be applied to some or all of these stars, with, perhaps, special attention to those for which data are available on interstellar calcium lines.

Summary.—Williams's system of classification of B-type stars by means of ratios of photometrically measured line intensities is briefly summarised. It is shown that this method may be adapted to unstandardised objective-prism spectra by visual estimation of similar intensity ratios.

Methods of estimation are explained and applied to 45 of the stars originally measured by Williams, with satisfactory results.

A simple and rapid method is described of measuring wing widths of lines by means of a masking wedge.

Widths measured in this way correlate well with the measured values of (total absorption)/(line depth), and are thus a good index of luminosity. The measures for *H* and *He* are plotted against absolute magnitudes as found by Williams, and give reduction curves which can be used for the determination of absolute magnitudes or parallaxes with greater accuracy than by previous spectroscopic methods.

Tables and diagrams are given illustrating the results obtained (classification and luminosity) for the 45 stars investigated.

The Norman Lockyer Observatory, Sidmouth :
1938 March 31.

A NEW SOLUBLE CASE OF THE EQUATION OF TRANSFER FOR A LINE FREQUENCY.

Max Krook, Isaac Newton Student, Cambridge University.

1. The mathematical theory of the formation of absorption lines in stellar atmospheres is usually made to depend on the variation with depth, of the ratio η_ν of line scattering coefficient to continuous absorption coefficient :

$$\eta_\nu = \frac{l_\nu}{k_\nu}. \quad (1)$$

On the basis of a definite theory for the continuous absorption coefficient k_ν , it is formally possible to calculate for given chemical constitution and effective temperature the distribution with depth of atoms of a particular element in a particular state of ionisation and excitation. Then, if at some level the number of such atoms per unit mass is N , the line scattering coefficient there is

$$l_\nu = N s_\nu, \quad (2)$$

where s_ν is the atomic scattering coefficient for radiation of frequency ν . In this way η_ν is calculable as a function of height in the atmosphere.

There are three optical depths which enter into discussions of the problem. These are :

(1) The optical depth τ in the continuous spectrum at the specified frequency :

$$\tau = \tau_\nu = \int_x^0 k_\nu \rho dx. \quad (3)$$

(2) The general optical depth $\bar{\tau}$ defined in terms of the mean absorption coefficient $\bar{k}$ averaged over all frequencies :

$$\bar{\tau} = \int_x^0 \bar{k} \rho dx. \quad (4)$$

It is this $\bar{\tau}$ in terms of which the temperature distribution is specified by the (approximate) relation

$$T^4 = T_e^4 \left(\frac{1}{2} + \frac{3}{4} \bar{\tau} \right). \quad (5)$$

(3) Many writers prefer to use the optical depth t_ν in the frequency ν of the line itself :

$$t_\nu = \int_x^0 (k_\nu + l_\nu) \rho dx. \quad (6)$$

In the present state of the theory it is usual to make the simplifying assumption that the ratio

$$n_\nu = \frac{k_\nu}{\bar{k}} \quad (7)$$

of the monochromatic to the mean, continuous absorption coefficient is independent of depth in the atmosphere. This amounts to taking k_ν to be the same function of ν for all layers. The relation between the optical depths τ and $\bar{\tau}$ is then

$$\tau = \tau_\nu = n_\nu \bar{\tau}. \quad (8)$$

The ratio η_ν can now be written

$$\eta_\nu = \frac{N \cdot s_\nu}{k_\nu} = \frac{N}{\bar{k}} \cdot \frac{s_\nu}{n_\nu}. \quad (9)$$

In (9) the factor $N/\bar{k}$ depends only on τ , whilst to a first approximation the factor s_ν/n_ν depends only on ν . n_ν varies only slowly with ν and may be taken as constant over a line; s_ν , on the other hand, varies rapidly with ν in the region of a line, and is responsible for the shape and the "scale" of the line contour. Collision broadening, Doppler effect, etc., influence η_ν only through its dependence on s_ν ; these effects may be regarded as approximately constant through the atmosphere, so that s_ν is appreciably independent of τ . (In the case of Stark effect the variation of s_ν with τ may have to be taken into account.) We shall write

$$\left. \begin{aligned} \frac{N}{\bar{k}} &= \psi(\tau) \\ \frac{s_\nu}{n_\nu} &= f(\nu) \end{aligned} \right\},$$

so that

$$\eta_\nu = f(\nu)\psi(\tau). \quad (10)$$

2. In terms of the optical depth τ in the neighbouring continuous spectrum, the equation of transfer for the frequency ν of a line is (neglecting collisions and fluorescence, *i.e.* $\epsilon = 0$)

$$\cos \theta \frac{dI_\nu}{d\tau} = (1 + \eta_\nu)I_\nu - \eta_\nu \mathfrak{J}_\nu - B_\nu. \quad (11)$$

This leads by the usual methods with the approximation $K_\nu = \frac{1}{3}\mathfrak{J}_\nu$ to the equations

$$\frac{dH_\nu}{d\tau} = \mathfrak{J}_\nu - B_\nu, \quad (12)$$

$$\frac{d\mathfrak{J}_\nu}{d\tau} = 3(1 + \eta_\nu)H_\nu. \quad (13)$$

Differentiating (12) and substituting from (13),

$$\frac{d^2 H_\nu}{d\tau^2} = 3(1 + \eta_\nu)H_\nu - \frac{dB_\nu}{d\tau}. \quad (14)$$

This second order differential equation for H_ν as a function of τ is formally equivalent to the equation for $\mathfrak{J}_\nu$ as a function of τ which is usually employed

in discussions of the problem. We use the form (14) for the reason that the desired results appear more readily in terms of it. Exact solutions of (14) have been given in only a few cases of η_ν as a function of depth. The solutions* for $\eta_\nu = \text{constant}$ and $1/(1 + \eta_\nu) = \alpha_\nu + \beta_\nu \tau$ have been applied fairly extensively. Spitzer† has also recently discussed the solutions for a functional form

$$1 + \eta_\nu = A(1 + D\tau)^s,$$

an expression involving three arbitrary constants for fitting to an observed distribution of $1 + \eta_\nu$.

The disadvantage of the functional forms cited above is that, except in the case $\eta_\nu = \text{constant}$, the numerical coefficients have to be redetermined for each frequency in the line. This is due to the fact that the assumptions of a functional form refer to $1 + \eta_\nu$ and not to η_ν directly, so that η_ν cannot be written in the required form (10). The position in the line should be controlled only by $f(\nu)$, whilst $\psi(\tau)$ remains invariant. To illustrate this argument we consider the Eddington formula‡ for two frequencies ν_1, ν_2 , and suppose the function fitted at ν_1 :

$$\frac{1}{1 + \eta_1} = \frac{1}{1 + f(\nu_1)\psi(\tau)} = \alpha_1 + \beta_1 \tau.$$

This leads, for frequency ν_2 , to

$$\frac{1}{1 + \eta_2} = \frac{f(\nu_1)}{f(\nu_2)} \cdot \frac{\alpha_1 + \beta_1 \tau}{1 + \left\{ \frac{f(\nu_1)}{f(\nu_2)} - 1 \right\} (\alpha_1 + \beta_1 \tau)},$$

which is not of the required functional form. Only by fitting a function directly to the observed variation of η_ν , or what is equivalent, of $\psi(\tau)$ can we obviate this difficulty.

3. A function satisfying this requirement and at the same time admitting of some elasticity in fitting to observed distributions by virtue of its containing three disposable constants is

$$\begin{aligned} \eta_\nu &= f(\nu) \{ \alpha + \beta e^{-\lambda \tau} \}, \\ \eta_\nu &= \alpha_\nu + \beta_\nu e^{-\lambda \tau} \end{aligned} \quad \left. \vphantom{\begin{aligned} \eta_\nu &= f(\nu) \{ \alpha + \beta e^{-\lambda \tau} \}, \\ \eta_\nu &= \alpha_\nu + \beta_\nu e^{-\lambda \tau} \end{aligned}} \right\} \quad (\lambda \geq 0). \quad (15)$$

The parameters α_ν and β_ν specify the position in the line and λ is the same for all points of the line. We shall as usual approximate to the Planck function B_ν by means of a linear function of the general optical depth $\bar{\tau}$, and thus by (8) a linear function also of the optical depth τ in the neighbouring continuous background.

$$B_\nu(\tau) = a_\nu + p_\nu \bar{\tau} = a_\nu + \frac{p_\nu}{n_\nu} \tau = a_\nu + b_\nu \tau. \quad (16)$$

* Cf. Eddington, *M.N.*, **89**, 620, 1929.

† *Ap. J.*, **87**, 1.

‡ Cf. Eddington, *M.N.*, **89**, 620, 1929.

Substituting (15) and (16) in (14) we obtain

$$\frac{d^2 H_\nu}{d\tau^2} = 3\{(1 + \alpha_\nu) + \beta_\nu e^{-\lambda\tau}\} H_\nu - b_\nu. \quad (17)$$

Put

$$\left. \begin{aligned} \frac{2\sqrt{3\beta_\nu}}{\lambda} e^{-\frac{1}{2}\lambda\tau} &= z_\nu \\ \frac{12(1 + \alpha_\nu)}{\lambda^2} &= n_\nu^2 \end{aligned} \right\}, \quad (18)$$

(17) then becomes

$$z_\nu^2 \frac{d^2 H_\nu}{dz_\nu^2} + z_\nu \frac{dH_\nu}{dz_\nu} - (z_\nu^2 + n_\nu^2) H_\nu = -\frac{4b_\nu}{\lambda^2}. \quad (19)$$

The operator applied to H_ν on the left-hand side of (19) is the familiar Bessel operator leading to the functions $I_n(z)$, $K_n(z)$ of complex argument.* The complete solution of (19) may now be written, dropping the suffix ν for convenience,

$$H = PI_n(z) + QI_{-n}(z) + \frac{4b}{\lambda^2} s_n(z), \quad (n \neq \text{an integer}), \quad (20)$$

$$H = PI_n(z) + QK_n(z) + \frac{4b}{\lambda^2} s_n(z), \quad (n = \text{an odd integer}), \quad (21)$$

$$H = MI_n(z) + NK_n(z) + \frac{4b}{\lambda^2} S_n(z), \quad (n = \text{an even integer}), \quad (22)$$

where P , Q and M , N are constants of integration and the particular integrals $s_n(z)$ and $S_n(z)$ are related to the Lommel two-index functions for real argument.† The reason why the case $n = \text{an even integer}$ is put on a different footing from the other cases is that the series expansion defining $s_n(z)$ is no longer valid for this case. We have instead as particular integral the polynomial $S_n(z)$ of degree n in $\left(\frac{1}{z^2}\right)$.

$$s_n(z) = \frac{1}{n^2} \left\{ 1 + \frac{z^2}{(2^2 - n^2)} + \frac{z^4}{(2^2 - n^2)(4^2 - n^2)} + \dots \right\}, \quad n \neq \text{an even integer}, \quad (23)$$

$$S_n(z) = \frac{1}{z^2} \left\{ 1 + \frac{(2^2 - n^2)}{z^2} + \frac{(2^2 - n^2)(4^2 - n^2)}{z^4} + \dots \right\}, \quad n = \text{an even integer}. \quad (24)$$

By virtue of the type of boundary conditions to be applied we are able to treat the two cases (20) and (21) together. (22), however, has to be discussed separately. These boundary conditions are as follows:—

$$(a) \ H \text{ must remain finite as } \tau \rightarrow \infty, \text{ i.e. as } z \rightarrow 0, \quad (25)$$

$$(b) \ \mathcal{F} = 2H \text{ for } \tau = 0, \text{ i.e. for } z = z_0 = \frac{2\sqrt{3\beta_\nu}}{\lambda}. \quad (26)$$

* Cf. Watson's Bessel Functions.

† Watson's Bessel Functions.

4. Case I: $n = \text{an even integer}$. Here, since both $I_{-n}(z)$ and $K_n(z)$ tend to infinity as $z \rightarrow 0$, whilst $I_n(z)$ and $s_n(z)$ remain finite, we must have in (20) and (21) that $Q \equiv 0$. In the solution

$$H(z) = PI_n(z) + \frac{4b}{\lambda^2} s_n(z)$$

the constant P is to be evaluated from the boundary condition (b) above. We have from (12) and (18),

$$\mathcal{J} = a + b\tau + \frac{dH}{d\tau},$$

i.e.

$$\mathcal{J}(z) = a + b\tau - \frac{1}{2}\lambda z \frac{dH}{dz}.$$

Determining P in this way from the relation $\mathcal{J}(z_0) = 2H(z_0)$, we obtain for the emergent flux ($z = z_0$)

$$H(z_0) = \frac{aI_n(z_0) - \frac{4b}{\lambda^2} \cdot \frac{1}{2}\lambda z_0 \{I_n(z_0)s'_n(z_0) - I'_n(z_0)s_n(z_0)\}}{2I_n(z_0) + \frac{1}{2}\lambda z_0 I'_n(z_0)},$$

where the dashes denote differentiation with respect to z . The intensity expressed as ratio of the emergent flux in the frequency of the line to the emergent flux in the continuous background * is then

$$r_\nu = \frac{a_\nu - \frac{4b_\nu}{\lambda^2} \sqrt{3\beta_\nu} \left\{ s'_{n_\nu} \left(\frac{2\sqrt{3\beta_\nu}}{\lambda} \right) - s_{n_\nu} \left(\frac{2\sqrt{3\beta_\nu}}{\lambda} \right) F \left(\frac{2\sqrt{3\beta_\nu}}{\lambda} \right) \right\}}{2 + \sqrt{3\beta_\nu} F \left(\frac{2\sqrt{3\beta_\nu}}{\lambda} \right)} \cdot \frac{2 + \sqrt{3}}{a_\nu + b_\nu/\sqrt{3}}, \quad (27)$$

where

$$F(z) = I'_{n_\nu}(z)/I_{n_\nu}(z). \quad (28)$$

Since the functions $s_n(z)$ and their derivative $s'_n(z)$ are not tabulated they have to be evaluated by summing the series (23) and the one derived from it by differentiation. These series converge rapidly, however, when z_0 is not too large, *i.e.* to within moderate distances from the line centre. For the greater part of the contour it is sufficient to take only a few terms of the series. The functions $I_n(z)$, $K_n(z)$ are tabulated for a number of values of n in various tables, *e.g.* Watson's Bessel Functions, British Association tables, etc. For $n = \text{half-integral values}$ they are expressible simply in terms of exponential functions.

5. Case II: $n = \text{an even integer}$. Here as $\tau \rightarrow \infty$, *i.e.* $z \rightarrow 0$, both $K_n(z)$ and $S_n(z) \rightarrow \infty$ as $1/z^{2n}$:

$$\left. \begin{aligned} S_n(z) &\sim \frac{1}{z^2} + \frac{(2^2 - n^2)}{z^4} + \dots \\ K_n(z) &\sim (-1)^{\frac{1}{2}n-1} \cdot n \cdot S_n(z) \text{ as } z \rightarrow 0 \end{aligned} \right\}.$$

* The emergent flux in the continuous background is evaluated in the usual way with $\eta_\nu \equiv 0$ in (14).

Since by (25) H must remain finite as $z \rightarrow 0$, we have

$$N = (-1)^{\frac{1}{2}n} \cdot \frac{4b}{n\lambda^2}.$$

The constant M in the solution

$$H(z) = MI_n(z) + \frac{4b}{\lambda^2} \left\{ (-1)^{\frac{1}{2}n} \cdot \frac{1}{n} K_n(z) + S_n(z) \right\}$$

is again to be determined by the second boundary condition (26). Evaluating M in this way, we obtain for the intensity ratio in the line :

$$r_\nu = \frac{2 + \sqrt{3}}{a_\nu + b_\nu/\sqrt{3}} \cdot \frac{1}{2 + \sqrt{3}\beta_\nu F(z_0)} \left\{ a_\nu - \frac{4b_\nu}{\lambda^2} \sqrt{3\beta_\nu} \left[\frac{(-1)^{\frac{1}{2}n}}{n} \cdot K'_n(z_0) + s'_n(z_0) \right. \right. \\ \left. \left. - \left(\frac{(-1)^{\frac{1}{2}n}}{n} K_n(z_0) + s_n(z_0) \right) F(z_0) \right] \right\}, \quad (29)$$

where

$$\left. \begin{aligned} F(z_0) &= I'_n(z_0)/I_n(z_0) \\ z_0 &= \frac{2\sqrt{3\beta_\nu}}{\lambda} \end{aligned} \right\}. \quad (30)$$

6. This completes the discussion of the possible cases corresponding to the law $\eta_\nu = \alpha_\nu + \beta_\nu e^{-\lambda\tau}$ for the variation of η_ν . It is also a simple matter to treat composite configurations of the type

$$\left. \begin{aligned} \eta_\nu &= \alpha_{1\nu} + \beta_{1\nu} e^{-\lambda_1\tau}, & 0 \leq \tau \leq \tau_1, & & (\lambda_1 \geq 0) \\ \eta_\nu &= \alpha_{2\nu} + \beta_{2\nu} e^{-\lambda_2\tau}, & \tau_0 \geq \tau_1, & & (\lambda_2 > 0) \end{aligned} \right\}$$

by this method. The expressions are, however, apt to become rather cumbersome in this case, and will not be reproduced here.

Summary.—Solutions are given for the equation of transfer in the frequency ν of a line when the ratio $\eta_\nu = f(\nu)\{\alpha + \beta e^{-\lambda\tau}\}$. This functional form has the advantage that a displacement from one part of the line to another leaves the form invariant, position in the line being controlled only by the factor $f(\nu)$. Also the approximation $B_\nu = a_\nu + b_\nu\tau$, linear in the optical depth in the *continuous* spectrum, is used.

Gonville and Caius College,

Cambridge :

1938 March 31.

OCCULTATIONS OBSERVED AT THE ROYAL OBSERVATORY, CAPE OF GOOD HOPE, IN 1937.

(Communicated by H.M. Astronomer)

The occultations were reduced by the method and with the constants given in the *Supplement to the Nautical Almanac for 1938*, except Nos. 2 and 17, which were reduced in the same manner as in previous years (see *M.N.*, 96, 808, 1936).

Observers :

Dr. J. Jackson, J.
Dr. R. H. Stoy, S.
Mr. A. J. Wilkin, A. W.

Co-ordinates of instruments used :

Instrument	Longitude			Latitude			Log $\rho \cos \phi'$	Log $\rho \sin \phi'$
	^h	^m	^s	[°]	[']	["]		
7-inch heliometer	1	13	54.65 E.	- 33	56	1.4	9.91937	9.74434 ⁿ
10- „ equatorial			54.70			7.8	6	6
18- „ „			54.57			6.8	6	6
8- „ „			54.57			6.8	6	6
7- „ „			54.64			7.0	6	6

The 7-inch equatorial is now mounted in the dome which formerly contained the 6-inch.

A correction of +3".0 has been added to the Moon's mean longitude as given in the *Nautical Almanac*.

An analysis of the equations of condition gives the corrections to the orbital longitude ΔL and latitude ΔB :

Mean Epoch	ΔL	Wt.	ΔB	Wt.
1937.51	+2".59	30.7	-1".28	14.2

Using a value of ΔL varying uniformly from +2".80 at 1937.0 to +2".40 at 1938.0 and $\Delta B = -0".50$, the probable error of a single observation is $\pm 0".78$.

No.	Date	Star	Mag.	Phase	Instrument	Obs.	U.T.		
							h	m	s
1	Jan. 3	431 B Leonis	6.2	R.D.	Heliom.	A. W.	1	0	8.68
2	" 17	B.D. +5° 5236	8.4	D.D.	10 inch	S.	18	46	31.44
3	" 20	μ Arietis	5.7	D.D.	10 "	S.	20	1	27.72
4	" 20	"	"	"	Heliom.	A. W.	20	1	27.42
5	" 22	95 Tauri	6.2	D.D.	"	A. W.	18	49	29.16
6	Feb. 17	B.D. +21° 447	6.9	D.D.	18 inch	J.	18	7	16.42
7	" 17	"	"	"	Heliom.	A. W.	18	7	16.22
8	" 20	B.D. +22° 1352	6.8	D.D.	10 inch	S.	19	56	55.63
9	" 20	"	"	"	Heliom.	A. W.	19	56	55.03
10	" 21	61 Geminorum	5.9	D.D.	10 inch	S.	18	22	38.63
11	" 21	"	"	"	Heliom.	A. W.	18	22	38.93
12	" 21	B.D. +20° 1822	7.1	D.D.	10 inch	S.	21	15	23.46
13	Mar. 19	6 Geminorum	6.3	D.D.	10 "	S.	17	43	43.24
14	" 19	"	"	"	Heliom.	A. W.	17	43	43.04
15	Apr. 16	B.D. +21° 1426	6.7	D.D.	10 inch	S.	17	33	31.57
16	" 16	"	"	"	Heliom.	A. W.	17	33	31.57
17	" 16	B.D. +21° 1437	8.0	D.D.	"	A. W.	18	58	11.87
18	" 19	B.D. +9° 2226	6.8	D.D.	"	A. W.	21	33	57.42
19	" 26	ι Libræ	4.7	R.D.	"	A. W.	22	4	10.05
20	" 30	121 B Sagittarii	5.9	R.D.	"	A. W.	22	12	22.42
21	July 14	B.D. -9° 3569	6.6	D.D.	"	A. W.	19	56	46.12
22	" 17	ι Libræ	4.7	D.D.	"	A. W.	16	9	31.98
23	Aug. 12	40 H Virginis	5.1	D.D.	10 inch	S.	19	57	14.20
24	" 16	B.D. -21° 4659	6.6	D.D.	8 "	J.	20	48	56.28
25	" 16	"	"	"	10 "	S.	20	48	56.48
26	" 28	65 Arietis	5.9	R.D.	7 "	S.	3	2	25.92
27	" 29	ν Tauri	4.4	R.D.	7 "	S.	3	58	36.83
28	" 29	72 Tauri	5.4	R.D.	7 "	S.	4	31	28.54
29	Sept. 15	B.D. -17° 5746	7.1	D.D.	Heliom.	A. W.	18	5	8.77
30	Oct. 7	B.D. -18° 3997	6.8	D.D.	7 inch	S.	17	36	11.25
31	" 7	"	"	"	Heliom.	A. W.	17	36	11.65
32	" 9	B.D. -21° 4449	6.8	D.D.	7 inch	S.	18	30	48.45
33	" 9	"	"	"	Heliom.	A. W.	18	30	47.85
34	" 11	B.D. -20° 5223	6.8	D.D.	7 inch	S.	19	41	28.50
35	" 13	16 B Capricorni	6.2	D.D.	7 "	S.	19	55	52.50
36	" 13	β Capricorni	3.2	D.D.	7 "	S.	20	7	17.22
37	" 13	"	"	"	Heliom.	A. W.	20	7	17.12
38	Nov. 12	187 B Aquarii	6.3	D.D.	7 inch	S.	21	19	33.18
39	" 12	"	"	"	Heliom.	A. W.	21	19	33.18
40	Dec. 9	138 B Aquarii	6.4	D.D.	"	A. W.	19	33	47.21
41	" 9	B.D. -5° 5738	7.3	D.D.	"	A. W.	20	23	24.26
42	" 10	3 Piscium	6.4	D.D.	7 inch	S.	19	48	11.39
43	" 10	"	"	"	Heliom.	A. W.	19	48	11.39
44	" 13	101 Piscium	6.2	D.D.	7 inch	S.	19	53	8.89
45	" 13	"	"	"	Heliom.	A. W.	19	53	9.09
46	" 19	3 Cancri	5.8	R. D.	"	A. W.	22	7	42.97

Equations of Condition

1	-0.3 = + 4.5 $\Delta\alpha$ + .95 $\Delta\delta$ = - .10 ΔL + .99 ΔB
2	-0.6 = + 8.8 + .81 = + .86 + .52
3-4	+1.5 = +13.2 - .35 = + .83 - .56
5	+0.9 = +12.4 - .42 = + .88 - .47
6-7	-0.2 = +13.4 + .26 = +1.00 + .08
8-9	+0.7 = + 4.3 - .95 = + .43 - .90
10-11	+1.4 = +10.5 - .67 = + .87 - .49
12	+0.4 = - 0.2 -1.00 = + .21 - .98
13-14	+2.4 = +10.4 - .66 = + .81 - .58
15-16	+0.9 = + 8.2 - .81 = + .71 - .70
17	+1.0 = + 8.8 - .78 = + .75 - .66
18	+1.7 = + 2.8 - .98 = + .53 - .85
19	-0.1 = -11.9 + .53 = - .93 + .36
20	+0.9 = - 7.0 - .87 = - .62 - .79
21	+1.0 = + 8.4 - .83 = + .82 - .58
22	-0.7 = +13.6 - .27 = +1.00 - .08
23	-0.2 = +13.1 - .41 = + .99 - .15
24-25	-1.8 = +13.0 + .36 = + .95 + .32
26	-2.6 = - 3.3 - .97 = - .41 - .91
27	-1.7 = -13.6 - .16 = -1.00 - .08
28	+0.9 = -12.6 + .41 = - .88 + .48
29	-1.5 = + 4.5 + .95 = + .53 + .85
30-31	-2.2 = +14.0 + .16 = + .94 + .34
32-33	-1.7 = +12.9 + .38 = + .92 + .40
34	-0.6 = +14.1 - .02 = + .93 - .37
35	-2.0 = +14.2 - .20 = + .89 - .46
36-37	+0.4 = +13.8 - .31 = + .83 - .56
38-39	-0.7 = +14.6 + .21 = + .99 - .16
40	+0.1 = +14.4 + .26 = + .99 - .11
41	-0.7 = +11.4 + .64 = + .95 + .32
42-43	-1.4 = +10.8 + .69 = + .93 + .37
44-45	-1.4 = + 3.1 + .98 = + .50 + .87
46	+1.7 = -10.6 + .67 = - .88 + .47

2. App. place from Cape observations, 1937, 23^h 51^m 39^s.88, +5° 45' 29".0.
 17. App. place from Abbadia 1900 and Albany 1910, 6^h 53^m 5^s.37, +21° 2' 57".8.
 22. Daylight.
 24. Seeing poor.
 25. " "
 33. Sluggish disappearance.

OCCULTATIONS OF STARS BY THE MOON, OBSERVED AT THE NIZAMIAH OBSERVATORY, HYDERABAD, DURING THE YEAR 1937.

(Communicated by the Director)

No.	Star	Mag.	Phase	Date	U.T.		
					h	m	s
				1937			
1	101 Piscium	6.2	D	Jan. 19	16	25	57.7
2	284 B Tauri	6.0	D	22	17	7	41.2
3	B.D. + 23° 1007	6.5	D	23	18	34	46.6
4	394 B Tauri	6.1	D	23	19	22	3.9
5	h Leonis	5.3	D	27	19	16	58.1
6	h Leonis	5.3	R	27	20	14	26.7
7	ζ Geminorum	3.7	D	Mar. 20	15	23	41.1
8	Abbadia 4003	8.2	D	20	15	25	31.0
9	3 Cancri	5.8	D	21	15	39	9.3
10	21 G Sag	5.7	R	Apr. 2	23	37	18.1
11	δ Scorpii	2.5	R	27	22	38	31.7
12	237 B Leonis	6.3	D	May 18	14	44	17.1
13	55 Leonis	6.0	D	18	17	20	26.0
14	B.D. + 1° 2502	6.9	D	18	17	50	18.3
15	B.D. - 7° 3409	6.7	D	20	17	22	8.5
16	19 Piscium	5.3	R	July 27	22	6	23.5
17	B.D. - 22° 4336	7.4	D	Aug. 16	14	16	13.6
18	κ Piscium	4.9	R	23	18	34	23.3
19	27 G Capricorni	6.2	D	Sept. 16	14	42	47.6
20	B.D. - 22° 4377	9.5	D	Oct. 10	13	24	26.1
21	B.D. - 21° 5025	7.4	D	11	15	13	12.3
22	247 B Tauri	5.7	R	22	16	9	21.0
23	Hyd - 17° 60198	..	D	Nov. 9	13	15	51.6
24	Washington 7464	8.8	D	9	13	28	55.7
25	Washington 7466	9.3	D	9	13	43	53.0
26	Washington 7468	9.0	D	9	13	49	21.9
27	Washington 7467	9.4	D	9	14	4	48.8
28	Washington 7473	9.0	D	9	14	25	12.2
29	B.D. - 17° 5780	9.4	D	9	14	28	6.0
30	Washington 7469	8.6	D	9	14	31	17.7
31	44 Aquarii	5.8	D	12	12	43	45.9
32	B.D. - 1° 4393	7.4	D	13	13	36	17.6
33	ι Tauri	4.7	R	19	15	22	17.1
34	B.D. - 20° 5268	6.9	D	Dec. 5	13	22	21.3
35	B.D. - 7° 5685	9.8	D	9	14	11	50.9
36	30 Aquarii	5.6	D	9	14	42	12.9
37	B.D. - 7° 5689	9.5	D	9	14	56	20.2
38	B.D. - 7° 5691	9.5	D	9	15	0	6.4
39	B.D. + 16° 281	6.8	D	14	14	16	25.2

Notes

- No. 6 : slightly late.
- Nos. 12, 18 : seen through clouds.
- No. 13 : not instantaneous.
- No. 34 : observed with a 3-inch portable telescope.

Seven observations (No. 23 and Nos. 34 to 39) were made by T. P. Bhaskaran and the rest by M. K. Bappu. The instrument used was the 15-inch visual refractor, except in the case of No. 34, which was observed with a 3-inch portable telescope placed in the verandah of the observing house. The clock corrections were determined from observations with the transit instrument.

The reductions were carried out by M. V. V. Sastri and S. Aravamudan. As in the previous year, a correction of 0^h.0015 has been added to the observed time to allow for the error of 3" in the mean longitude of the Moon.

No.	χ	ρ	$\sigma' - \sigma$	$\cos (\chi - \rho)$	No.	χ	ρ	$\sigma' - \sigma$	$\cos (\chi - \rho)$
	[°] [']	[°] [']	["]			[°] [']	[°] [']	["]	
1	96 41	71 26	-1.4	+0.904	21	34 56	82 41	-1.6	+0.672
2	97 56	86 37	-1.8	+0.980	22	322 6	85 37	+0.4	-0.552
3	83 24	93 8	-0.3	+0.986	23	43 3	76 6	+0.2	+0.838
4	92 39	93 20	+0.4	+1.000	24	84 28	76 4	0.0	+0.989
5	176 32	111 5	-1.3	+0.416	25	98 49	76 2	-4.9	+0.922
6	254 53	111 11	+4.3	-0.806	26	90 49	76 1	-1.4	+0.967
7	67 18	100 33	-1.4	+0.836	27	143 43	75 59	+0.8	+0.379
8	60 10	100 34	+1.3	+0.762	28	65 2	75 56	+0.3	+0.982
9	90 55	105 8	-1.9	+0.969	29	58 51	75 55	-1.5	+0.956
10	307 44	85 10	+1.3	-0.736	30	20 59	75 55	-0.9	+0.574
11	234 32	96 28	+0.7	-0.744	31	55 4	68 50	-0.7	+0.971
12	144 57	112 35	-0.6	+0.845	32	59 38	68 11	-0.1	+0.989
13	81 18	112 37	-1.6	+0.854	33	239 51	89 1	+0.5	-0.873
14	129 38	112 37	-0.7	+0.956	34	96 5	81 2	-0.8	+0.966
15	77 22	111 15	-1.3	+0.830	35	12 37	69 10	-2.1	+0.551
16	196 1	67 45	+1.3	-0.619	36	88 31	69 8	-1.8	+0.943
17	79 28	89 6	-1.4	+0.986	37	66 13	69 7	-1.8	+0.999
18	221 38	67 42	+1.6	-0.898	38	72 59	69 7	+0.4	+0.998
19	50 1	73 26	-1.5	+0.918	39	39 28	75 26	-1.9	+0.809
20	84 50	87 40	+2.5	+0.999					

The mean places for 1937.0 of thirteen stars in the above list are given in the following table. The positions have been computed from the measures in the Carte du Ciel Catalogues for use in reductions.

No.	Star	Mag.	R.A. (1937·0)	Declination (1937·0)	Authority
8	Abbadia 4003	8·2	^h ^m ^s 7 0 21·38	[°] ['] ["] +20 41 25·4	
20	B.D. - 22° 4377	9·5	17 32 39·67	-22 18 58·4	Hyderabad, -22° 57656 ; -23° 65492.
23	Hyd - 17° 60198	..	19 47 32·25	-17 18 14·8	Hyderabad, -17° 60198 ; -18° 59006.
24	Washington 7464	8·8	19 48 8·06	-17 25 46·5	
25	Washington 7466	9·3	19 48 26·07	-17 27 8·5	
26	Washington 7468	9·0	19 48 34·07	-17 24 10·1	
27	Washington 7467	9·4	19 48 28·67	-17 33 33·1	
28	Washington 7473	9·0	19 49 16·13	-17 11 38·4	
29	B.D. - 17° 5780	9·4	19 49 16·84	-17 9 43·9	Hyderabad, -17° 60180 ; -18° 58987.
30	Washington 7469	8·6	19 48 50·03	-17 2 55·3	
35	B.D. - 7° 5685	9·8	21 58 30·15	- 6 41 21·3	San Fernando, 21 ^h 56 ^m , -7°, No. 69 ; 22 ^h 0 ^m , -6°, No. 12.
37	B.D. - 7° 5689	9·5	22 0 11·71	- 6 40 56·5	San Fernando, 21 ^h 56 ^m , -7°, No. 96 ; 22 ^h 0 ^m , -6°, No. 39.
38	B.D. - 7° 5691	9·5	22 0 19·55	- 6 41 50·7	San Fernando, 21 ^h 56 ^m , -7°, No. 102 ; 22 ^h 0 ^m , -6°, No. 44.

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